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The design of a flexible bus line plan

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ABSTRACT

A conventional bus service has a line plan that consists of a set of lines with fixed routes designed to operate under normal conditions. Thus, the performance of the service depends on how stable those conditions are. This paper extends the traditional concept of a line plan to provide a high-quality service, even in case of announced events. We introduce the concept of a *Flexible Line Plan* where some of the usual lines are replaced with alternative lines that improve the service when the conditions change. Adaptations should only be made to a subset of the lines, ensuring that the changes minimize the inconvenience for the regular passengers and, at the same time, improve the total travel time. In this study, we present a heuristic called *Alternative Line Heuristic* (ALH), based on Genetic Algorithms, which automatically designs these *alternative lines* in a three-step process. The results of this approach are tested on both benchmark and real-sized networks and clearly show how our approach contributes to improving the bus service performance.

1. Introduction

One of the aims of public bus services is to transport people from different origins to destinations as fast as possible. In many cases, these services are organized based on fixed lines and fixed timetables to simplify their operation. Based on that information, regular passengers know which bus line to take, where to transfer if required, and how to estimate their travel times. This service is typically obtained by a design process that takes as input a set of parameters such as the expected demand and congestion, the number of available bus units, the location of bus stops, etc. These parameters describe the *usual* (average) *situation*, which is expected to occur during the operation of the service. For that usual situation, an efficient line plan is designed, trading of operational costs and service quality, typically modeled by the total passenger travel time.

In this paper, we focus on improving the quality of the bus service in case of recurring events, which we call *unusual situations*. These could be weekly markets, sports games in a stadium or festivities in the city center. These events do not occur on a daily basis and therefore should not be permanently included in the usual line plan, but they are frequent enough so that they cannot be completely ignored, especially since they can be anticipated. These events typically attract many people, leading to extra demand towards and from the venue, but they also produce an inevitable congestion in the surrounding area. Ideally, the extra users should be offered a better service towards the congested area. Moreover, the users whose routes partially coincide with the areas disturbed by the event, should be able to avoid the area to minimize delays. Furthermore, for all other users, the service must remain as unaffected as possible.

This paper proposes a new concept to improve the performance of the bus service, also during an unusual situation. The concept starts from the current line plan, developed for the usual situation. Then, a limited number of lines is modified to meet the particularities of the unusual situation, while keeping the regular service for the other passengers unaffected. Due to exchanging some lines of the usual line plan for *alternative lines,* the new line plan outperforms the usual one during the event. We call this concept a *Flexible Line Plan*. Once the unusual situation is over, the usual line plan is used again.

Since some passengers will be faced with changing bus lines, they

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must be properly informed and guided. We will assume, for now, that travelers can be well informed and are willing to accept the changes. Hence, this paper focuses on exploring how alternative bus lines could be designed and whether this concept of *Flexible Line Plan* is helpful.

Designing alternative lines will require the design, evaluation, and comparison of different possible line plans. Hence, it should be noted that line planning is a very complex optimization problem due to the large number of connections between any pair of stops, which generates a combinatorial explosion of the number of possible bus lines. Moreover, the smallest change in an individual line might lead to many changes in trips of different passenger groups through the network. Therefore, when the passenger travel time is included in the objective function, long computation times are required and only (very) small instances are typically considered (Duran-Micco et al., 2019; Iliopoulou et al., 2019).

This paper contributes to the state of the art by presenting not only the concept of the *Flexible Line Plan*, but also an automated technique to design it. This technique will also be compared to two other automated techniques. Furthermore, we will show the potential benefits of the *Flexible Line Plan*, by applying our approach to the bus service of both a well-known benchmark instance and the city of Cuenca in Ecuador. When evaluating the potential of this *Flexible Line Plan* concept, many questions will arise, such as: How well would a service perform with a few alternative lines during an unusual situation? How to design the *Flexible Line Plan*? How many lines should change? Is it possible to design alternative lines that are similar to the lines they replace so that users do not face major changes? Are the alternative lines really an efficient way of dealing with recurring events? We answer these questions in the remainder of this paper.

It is worth mentioning that our approach clearly differs from robust line planning, where a single line plan should have an acceptable performance during the usual situation and during a set of unusual situations, without making changes when new events happen. Our *Flexible Line Plan* makes a limited number of changes, as soon as an event is announced so that a new line plan is used during the unusual situation. Still, the changes do not produce an optimal line plan for the unusual situation, since that would require to completely change the regular service, which would be unacceptable for many regular users.

The *Flexible Line Plan* offers an alternative for the typical real-time control when cancelling, shortening, or rerouting bus lines. These ad-hoc decisions are typically based on a local analysis to avoid busy or closed areas in case of events and to control the extra travel time, but the additional demand is not considered. In contrast, in our approach the changes are planned much earlier and keeping both the global performance of the system and the additional demand in mind. By changing the route of a limited number of bus lines, the drawbacks and the opportunities caused by recurrent events should be addressed in the best possible way. In this paper, only changes in the line plan are considered, whereas the timetable with the exact timing of all the buses is assumed to remain the same.

The rest of this paper is organized as follows. In Section 2, the related literature is discussed to situate this research. Later, in Section 3, the *Flexible Line Plan* concept is discussed in detail. We present the Genetic Algorithm used to design the line plans in this study and our methodology to implement and design the alternative lines in Section 4. In Section 5, several experiments demonstrate the benefits of this new concept and Section 6 concludes the paper.

2. Literature review

This literature study starts by discussing the state of the art concerning the typical line planning problem. Then we will compare flexible line planning with robust line planning and other related concepts.

2.1. Line planning problem

The study of Ceder and Wilson (1986), formally places the design of the line plan in the context of the "Public bus planning process". This is a 5-step process, beginning by line planning, then setting the frequencies, the development of the timetable, the definition of the crew schedule and at last, the vehicle schedule. During line planning, the number of bus lines, and the order in which each line visits different stops is decided. This step is also known as the network design problem (Iliopoulou et al., 2019).

One of the first studies to define bus lines corresponds to Patz (1925), who proposes an iterative algorithm to generate routes based on the bus capacity and the demand. The objective function attempts to minimize the number of empty seats. Later, Lampkin and Saalmans (1967), propose an algorithm that creates bus lines starting from an initial skeleton of a small number of nodes (bus stops); then, new nodes are inserted one by one according to the demand. The goal is to minimize the total travel time and the problem is constrained by the available fleet size.

In Mandl (1979), the author develops a method that starts with feasible routes and then the quality of the lines is improved iteratively, minimizing the total travel time of the passengers. This approach was tested in a small network that was later used by many others (e.g., Baaj & Hani, 1991; Chakroborty & Wivedi, 2002; Zhao et al., 2015; Duran-Micco et al., 2019), making it a benchmark network. Since the links, nodes, demand, and travel times for this network are available in the literature, we will also use that benchmark instance in this paper.

In the study of Ceder and Wilson (1986), the authors recommend a line plan based on the evaluation of lines at two different levels, considering the perspectives of both passengers and the operator. This leads to two conflicting goals in the Line Planning Problem (LPP). Typically, in an operator-oriented model, the costs are minimized while ensuring a certain level of quality for the passengers (e.g. Wan and Lo, 2003), while in a passenger-oriented model the quality of the service is maximized and the costs are limited (e.g. Yu et al., 2005; Fan and Mumford, 2010). One of the most appreciated indicators to evaluate the quality of a service is the passenger travel time (Van Oort, 2011). Users strongly prefer shorter and more predictable travel times, as reported in a review study (Carrión & Levinson, 2012). Therefore, many researchers try to minimize the total travel time when designing a line plan (e.g. Baaj & Hani, 1991; Chakroborty & Wivedi, 2002; Borndörfer et al., 2008; Zhao et al., 2015). The total travel time is the summation of the in-vehicle and the waiting time of all passengers. In this paper, we also minimize the total travel time.

In the literature, the LPP uses several parameters and constraints as input, such as the stops, road infrastructure or the origin–destination matrix (Guihaire & Hao, 2008; Ibarra-Rojas et al., 2015). The fleet size and a maximal line length are input parameters widely used to include the constraints of the operator into the design, and together limit the number of available buses. All these aspects will also be considered in this paper.

Several methods are used to solve the LPP and an overview can be found in Iliopoulou et al. (2019) and Ibarra-Rojas et al. (2015). According to Ibarra-Rojas et al. (2015), discrete optimization approaches are classified into bi-level optimization methods, metaheuristic algorithms and methods that decompose into sequential sub-problems. Metaheuristics have been used in a number of studies to deal with the LPP in small and real-sized networks (e.g., Pattnaik et al., 1998; Chakroborty & Wivedi, 2002; Ngamchai et al., 2003; Tom & Mohan, 2003; Zhao & Zeng, 2006; Cipriani et al., 2012; Nayeem et al., 2014; Zhao et al., 2015; Schöbel, 2017; Goerigk & Schmidt, 2017; Owais & Osman, 2018; Duran et al., 2019; Jha et al., 2019; Mahdavi Moghaddam et al., 2019 Liu et al., 2020). The most popular metaheuristics are genetic algorithms (Holland, 1975), and simulated annealing. However, other techniques are also used, such as ant colony optimization, tabu search and memetic algorithms (Farahani et al., 2013). For our research, we designed and coded a passenger-oriented Genetic Algorithm, which mimics the principles of natural evolution, such as variation, selection, recombination, and mutation, in order to evolve to a high-quality line plan. Although some studies start from a predetermined and limited pool of feasible bus lines (Chakroborty & Wivedi, 2002; Cipriani et al., 2012; Pattnaik et al., 1998; Tom & Mohan, 2003), in this paper, the pool of feasible lines are generated from scratch. We will explain our algorithm in Section 4 and all the details and its performance are discussed in Avila-Ordóñez et al. (2018).

2.2. Related concepts

The terms "*flexible services*" and "*semi-flexible transit systems*" are used in previous studies to define a service that is particularly useful in areas with low demand (e.g., Chang & Schonfeld, 1991; Nourbakhsh & Ouyang, 2012; Chen & Nie, 2017; Myungseob et al., 2019). A survey on planning semi-flexible services shows a classification by flexibility level and outlines the future research (Errico et al., 2013). Buses with small capacities offer flexible routes to the passengers, picking them up from different places towards a common destination. Kim and Schonfeld (2011) show that combining flexible and conventional services produces lower operating costs compared to using the services separately. The study of Sayarsahd and Gao (2020), presents an optimization model to dynamically switch between fixed and flexible services, including a social wellfare function to estimate the benefits from the passengers' perspectives regardless the combination of these services. A recent method uses the sensing capacity of the Internet tools together with a passengers request system to design a spatio-temporal flexible service that improves the traditional one (Li et al., 2020). Our *Flexible Line Plan* differs from these flexible services since our line plan is based on a conventional service: only the routes of some bus lines are modified, the locations of the bus stops stay unchanged, zones with low and high demand are considered and the fleet is homogeneous.

A common approach to deal with the uncertainty in public transport during operations is the design and implementation of robust services (e.g., Liebchen et al., 2009; Cicerone et al., 2009; Fischetti & Monaci, 2009; Marín et al., 2009). A robust service is a fixed service designed beforehand, so that the consequences of disturbances are limited. The approaches are mostly oriented to railways and nearly always they focus most on robust timetabling. Our *Flexible Line Plan* allows changes to the lines to improve the bus service in unusual situations. During operations, the decisions regarding the changes in the lines are made based on the analysis of a specific unusual situation. Consequently, for each unusual situation, another set of alternative lines is designed.

The research of de Neufville (2011) encourages planners to design infrastructure networks for adaptability and proposes the concept of "*flexible design*". Through this approach, engineering systems are designed with sufficient flexibility to minimize the negative effects of non-considered situations. This approach has been tested in several case studies such as: parking-size decisions (Zhao & Tseng, 2003), crash avoidance systems (de Neufville et al., 2008), oilfields (Lin, 2008) or pipeline-based networks (Melese et al., 2017). This is a long-term strategy, while the operation of our alternative lines is decided during operations. Actually, in our approach, the existing infrastructure is an input parameter that restricts the design of the lines.

Emerging public transportation services have been presented in the last years, one of them is the Customized Bus System (Lyu et al., 2019). This concept identifies passengers sharing a similar origin and destination and designs an efficient route for them. The routes are designed to be as direct as possible and the locations for bus stops are defined during this process. The service operates as follows: a passenger requests the service using the Internet, then the system finds a line to meet the requirement. If no line is found, the request is stored to periodically evaluate whether new lines should be offered. The demand in our *Flexible Line Plan* is not estimated by an explicit request from the passengers but by the attraction of an announced event. Furthermore, the location of each bus stop is previously defined in the infrastructure.

Obviously, during operations the demand hardly remains constant over time. This fact has motivated researches on multi-period line planning. A multi-period planning is used in serveral fields, the first application focused on manufacturing (Holt et al., 1955). Sahin et al. (2020), present an integer programming solution to design a line plan by discretizating the planning horizon and characterizing the demand as function of the time. Each period has fluctations on demand so, the model selects, for each period a subset of (predefined) lines to operate, resulting in a tailored line plan. These multi-period line plans show promising results and provide flexibility to the bus services. However, this approach differs from our *Flexible Line Plan* in the following three aspects: (1) The congestion is related to the period and not to the event. Not necessarily, the length of an event will coincide with a period. (2) multi-period planning focuses on fluctuations in demand and congestion during a day. These are usually included in the usual average situation when designing a line plan and, (3) the changes in the line plan are designed based on a predefined set of lines. Whereas, in our approach a new line is specifically designed for the situation.

As stated earlier, operational changes in the bus lines are allowed for a *Flexible Line Plan*. The lines remain unchanged until an event disrupts the usual situation. This is the situation under normal conditions, typically aimed at optimizing the service during peak hour traffic and with peak demand. The alternative lines of a *Flexible Line Plan* are then used during unusual but expected situations that occur less frequently. An unusual situation occurs due to the extra demand and congestion expected during events such as music festivals, sport games, parades, etc. At last, there are other situations that are unknown in advance because we cannot predict their occurrence, for instance an accident. In these circumstances, having alternative line plans available, already familiar to the users, might be useful. However, these unexpected situations are not yet considered in this study. In the next section, the concept of a *Flexible Line Plan* is discussed in detail.

3. The concept of a Flexible line plan

We start this section with some related definitions and then introduce the concept of a *Flexible Line Plan*. A *Public Transportation Network (PTN)* is commonly defined as $PTN = (V, E)$, where V is the set of bus stops and E are the infrastructure links used by the bus lines, that is the service's graph. A line l is a simple path in the *PTN*, which is a path that does not contain repeated bus stops. The set of lines operated in a bus service is called the line plan L. Based on the line plan L, it is assumed that passengers select the fastest path between their origin and destination, which might include transfers.

Line planning requires input data to design a line plan, for instance, the streets available for public transport, the expected number of passengers, the fleet size, etc. These are the *input parameters* to describe the context of operation for the line plan to be designed. In literature, several input parameters are shown (e.g. Guihaire & Hao, 2008; Ibarra-Rojas et al., 2015), although each of them contributes to a better solution, only seven basic input parameters are selected according to the specific goals of this study. Table 1 presents the details of each parameter.

Table 1
Input Parameters for line planning.

Parameter	Description
infrastructure network (*IN*)	The bus stops and links available for the public transport. Some of the links in this network may not be selected in the *PTN*. The *PTN* is a subset of IN.
demand (*OD*)	A matrix with the number of passengers for each OD-pair. The origins and destinations are the bus stops defined by *IN*.
link time (*LT*)	A matrix with the time required to travel each link (length and speed are implicitly defined)
maximal line length (*ML*)	The maximum travel time for a bus line between its first and last stop
stop time ($\overline{ST}$)	The average time, in minutes, spent by the bus in a bus stop
fleet size (*FS*)	The maximum number of available buses for the service
transfer time ($\overline{TST}$)	A fixed penalty, in minutes, in the travel time for each transfer

The performance of a line plan is measured by the total travel time T:

$$T = \sum_{i=1}^{b}\sum_{j=1}^{b}(SP_{ij} + \mathrm{ST}_{ij} + \mathrm{TST}_{ij})OD_{ij} \quad (1)$$

where:

b: $|V|$, the number of bus stops.
SP_{ij}: the shortest paths travel time from a bus stop i to a bus stop.j
ST_{ij}: the stop time, $\overline{ST}$ times the number of bus stops on the shortest path between bus stop i and bus stop.j
TST_{ij}: the transfer time, $\overline{TST}$ times the number of transfers on the shortest path between bus stop i and bus stop.j
OD_{ij} : the demand between bus stop i and bus stop.j

Although usual and unusual situations have been discussed before, a formal definition is given now. A *situation* describes the context of operation for a bus service. A *usual situation* describes the expected context of the operation and their expected variations, for instance, the "normal" congestion during peak hours. These fluctuations are included to facilitate the service's management and design a user-friendly service. However, when the situation changes in such a way that it exceeds the expected variations, an *unusual situation* is considered. In this paper, the unusual situations correspond to massive events that substantially alter the conditions for the bus service. Formally, a situation s is defined by a set of values for the seven parameters introduced in Table 1.

$$s = \{IN, OD, \mathrm{LT}, \mathrm{ML}, \overline{ST}, \mathrm{FS}, \overline{TST}\} \quad (2)$$

The usual situation will be referred as s_0 and s_u will refer to the unusual situation. Any of the parameters could change from s_0 to s_u but only fluctuations on demand (OD) and travel times (LT) are investigated in this paper.

The usual line plan L_0 is designed to operate efficiently in s_0. A *Flexible Line Plan* adapts this plan to cope with the unusual situation s_u. Therefore, an adapted line plan L_u is designed, so that:

$$T^0_{L_0} < T^0_{L_u} \quad (3)$$

$$T^u_{L_u} < T^u_{L_0} \quad (4)$$

where:

$T^k_{L_j}$, is the total travel time T of a line plan $L_j$in a situation s_k

The adaptation should not be limited to rerouting some lines to prevent the increase in travel times of regular passengers but should also improve the service for passengers attracted by the event. The line plan L_u should always be a better solution than L_0 during unusual situation as is indicated in (4). This could even be the optimal line plan for that situation or close to it, but when it comes to getting user approval for route changes, unusual line plans are designed as adaptations of L_0. Thus, some of the usual lines will continue operating in the unusual situations. This is described by the following formulation:

$$L_0 = \{l_1, l_2, l_3, \cdots l_m\} \quad (5)$$

$$A_u = \{a_1, a_2, a_3, \cdots a_q\} \quad (6)$$

$$L_u = \mathrm{U} \cup A_u$$

$$|\mathrm{U}| + A_u = m$$

$$A_u > 0$$

where:

l_i and a_i are bus lines;
$m = |L_0|$, is the number of usual lines;
$\mathrm{U} \subseteq L_0$;
A_u is the set of alternative lines designed for situation s_u.

An unusual line plan L_u has a subset of the lines of L_0 and a set of new *alternative lines* designed to get time savings in the unusual situation as indicated in (7). Many of the usual lines should be in L_u (8), and the higher that number, the less inconvenience users will notice. Thus, during the design of alternative lines, the number of usual lines that are kept should be defined. This decision depends on the total number of usual lines, the number of lines the operator is willing to modify, and whether the technique used to design the alternative lines finds a feasible solution under these constraints. Furthermore, the alternative lines should be similar to those they replace, so that some fragments of the usual lines are still present to guarantee for more users that the service has only been adapted and not completely modified.

4. Methodology

4.1. Passenger-oriented genetic algorithm to design a line plan

This genetic algorithm (GA) is used when designing the line plans and it is also used in the automated generation of the *Flexible Line Plan*. Only the headlines of this method are discussed here, since all the details are available in Avila-Ordóñez et al. (2018). There, the high-quality performance of the algorithm is also illustrated based on experimental results for benchmark instances.

A genetic algorithm is an evolutionary method that evolves an initial population towards, hopefully, an optimal solution, also called individual (Holland, 1975). The GA has six main steps presented in Algorithm 1. The GA starts by reading input parameters and then creating a pool of feasible lines, after designing w simple paths for each OD-pair. The number of simple paths (w) is one of the GA input parameters and should be defined according to the size of the problem to be solved. The initial and the subsequent populations are sets of individuals who are potential feasible line plans. The initial population is created by designing n individuals of nl lines that are taken from the pool of lines. The algorithm verifies if the graph corresponding to the line plan is strongly connected so that travelling to and from every bus stop is always possible.

The evolving step includes an elitist process where the individuals in the population are sorted based on their T. Then, another elitist selection process (Affenzeller et al., 2009) selects the best individuals of the current population to populate the next one. The new population is completed with the newly created individuals from the application of genetic operators (crossover and mutation) on individuals of the current population. This iterative process is repeated until the stop criterion is fulfilled.

The genetic operators, used in this algorithm, were designed considering the particularities of the *LPP*. The parent selection for the crossover uses a ranking method based on T. By combining a pair of parents, a new individual is designed to populate the new generation. Two strategies were designed for crossover. The first method combines the lines of the parents, taking a random number of lines from one of them and completing the new individual with lines from the other. The second method combines parts of two lines, one coming from each parent, to design a new line. Of course, these lines should have a bus stop in common that will be used as a pivot to combine fragments of both lines. Another genetic operator is the mutation that randomly selects an individual and attempts to add or remove one bus stop to one of its lines.

The line plans designed with this algorithm were tested for the benchmark network, reaching values similar to those reported in the literature. Thanks to this evidence, in the rest of this document, they will be referred to as "optimal line plans".

Algorithm 1:Steps of the passenger-oriented GA

Start
1 Read Input Data
  Situation's Input Parameters
  Genetic Algorithm's Input Parameters
2 Create a Pool of Lines
  *For each **OD**-pair greater than 0*
    *Create **w** simple paths*
    Include the simple paths to the pool
*3 Create **n** individuals for the Initial Population*
  Create an empty individual
  Repeat Until the individual is strongly connected
    Select a suitable line from the Pool of Lines
    Include that line to the individual
  *Repeat Until **nl** is reached*
    Take a line from the Pool of Lines
    Include the line to the individual
  Include the new individual in the Initial Population
4 Population=Initial Population
5 Evolve while the stop criterion is not reached
  For each individual in the Population
    Calculate T (fitness value)
  Take the best individuals (smaller T) from the Population
  Copy those individuals to the New Population
  Complete the New Population
    Combine individuals from the Population by crossover or Mutate an individual
    Include new individuals to the New Population
  Population=New Population
6 Select the best individual in the Population
End

4.2. Designing a flexible line plan

Line planning is a strategic decision, traditionally taken at a stage prior to the operation of the service. The design of a Flexible Line Plan extends this stage to the operation as illustrated in Fig. 1. The usual line plan L_0 is early designed in the strategic level and the unusual line plan L_u is decided as soon as an event is announced during a special planning time within operation, the newly created line plan should be operated every time the event occurs.

In subsection 4.1, details were presented about a method to solve the *LPP* based on a Genetic Algorithm. The usual line plan L_0should be created with this or some other similar method to ensure decent performance. The methodology to generate the alternative lines for an unusual line plan is described below.

4.3. Designing alternative lines

This section is dedicated to the Alternative Lines Heuristic (ALH) illustrated in Fig. 2. The heuristic consists of three steps. In the first step, our GA is used to design an "optimal" unusual line plan O_u for an unusual situation s_u. The usual line plan L_0 is included in the initial population of individuals for designing O_u, increasing the chance that a subset of its lines is included in the solution, since they were not affected by the unusual situation. However, this will be determined by the GA. Furthermore, many new lines will be designed, either to directly offer a solution regarding the effects of the disturbances or indirectly due to the optimization process that the algorithm is carrying out.

In the second step, O_u is compared with L_0. A process was carefully designed to compare the line plans and to obtain relevant results to be used when deciding on the alternative lines. The process starts by identifying the similarity between each pair of lines, where one line belongs to $L_0$and another to O_u. For non-identical pairs, the process calculates the percentage of bus stops they share and saves it in the matrix E where the rows and columns are the L_0 and O_u lines correspondingly. The pairs of lines that are identical (same bus stops and same order) are also identified, contributing to reduce the computational time to solve the problem, only focusing on the lines that are different.

Later, the path and the travel time for each OD-pair are calculated for the unusual situation, using both L_0 and O_u. During these calculations, the OD-pairs for which the travel time is reduced while using O_u are recorded, together with the bus lines that enable these savings. These bus lines could be efficient alternative lines and therefore they are considered as "potential alternative lines". These lines are saved in the set P.

The last step focuses on adapting L_0 based on the insights found in the second step. Algorithm 2 presents the details of this process. The algorithm reads the input data and the parameters: number of alternative lines, number of potential lines and number of attempts (*numAlternativeLines, numPotentialLines and numAttempts*), all of them are decided beforehand. Then, a copy of L_0 is made and it is called L. The algorithm uses the set P to select only the best *numPotentialLines* based on the savings that they produced. This subset is called P_1 and its lines are saved in a stack where the line that generates the greatest time savings is always on top. The algorithm will try to include a subset of *numAlternativeLines* from the top of P_1 (4.1 of Algorithm 2), by looking for the *numAttempts* lines in L that most closely resemble each line (4.2 of Algorithm 2), the algorithm uses the matrix E for this decision. Obviously, the goal is getting the greater benefits through subtle changes in the lines that are usually operated. A change is possible as long as L remains feasible and the T is improved for s_u, otherwise the algorithm will deal with the second most similar line and so on. When the attempts are exhausted, the algorithm tries with the next selected potential alternative line and proceeds as indicated. When a line produces a feasible solution and improves the T then the line is included in L as an alternative line. An iteration of 4.1 of Algorithm 2 may not produce a feasible solution so the algorithm removes the top line of P1 and repeats. This process is done *numAttempts* times. At the end, the algorithm returns the best L_u of all the attempts. In Section 5, the methodology is applied, and quantitative results are shown.

Algorithm 2. Selecting Algorithm.

4.4. Other techniques for designing alternative lines

ALH is a way of designing alternative lines, but it is certainly not the only possible way. In this paper two additional techniques are proposed, designed, implemented, and tested. The results of these are compared with those obtained by ALH in section 5.

The first approach is a modified version of the Genetic Algorithm called GA-P. This approach penalizes the individuals according to how different they are from L_0 and it calculates the total travel time as follows:

$$T_p = T + pq \tag{10}$$

where:

T: is the Total Travel Time (defined in Section 3).
p: the penalty for changes.
q: is the number of lines that are not in.L_0

The second is GA-F, it is also a modified version of GA which guarantees that a fixed number of lines from L_0 is present in each solution. This subset of lines can be different for each solution and the line plan is completed with alternative lines. Table 2 shows a summary of the techniques and their main similarities and differences.

Start
1 Read Input Data
    Situations: s_0 and s_u
    Line plans: L_0 and O_u
    Previous data: P and E
    Parameters: numAlternativeLines, numPotentialLines and numAttempts
2 Make a copy of L_0 in L
3 Create P_1 by selecting the most efficient numPotentialLines lines from P
4 Select the alternative lines
    minT=Inf
    4.1 Repeat Until numAttempts is reached
        Increase by 1 the number of attempts
        Pick up numAlternativeLines from P_1
        4.2 For each of the selected potential alternative lines
            4.3 Repeat Until numAttempts is reached
                Increase by 1 the number of attempts
                Find in E the most similar usual line for the current line
                Replace in L the usual line with the current line
                Is L no longer feasible?
                    Undo the last replacement
                    Mark as used the most similar line
                else
                    Break from the loop 4.3
        Is L feasible?
            Is $T_L^u < T_{L_0}^u$?
                Is $T_L^u < minT$?
                    $minT = T_L^u$
                    $L_u = L$
        $L = L_0$
        Remove the top of P_1
5 Return L_u

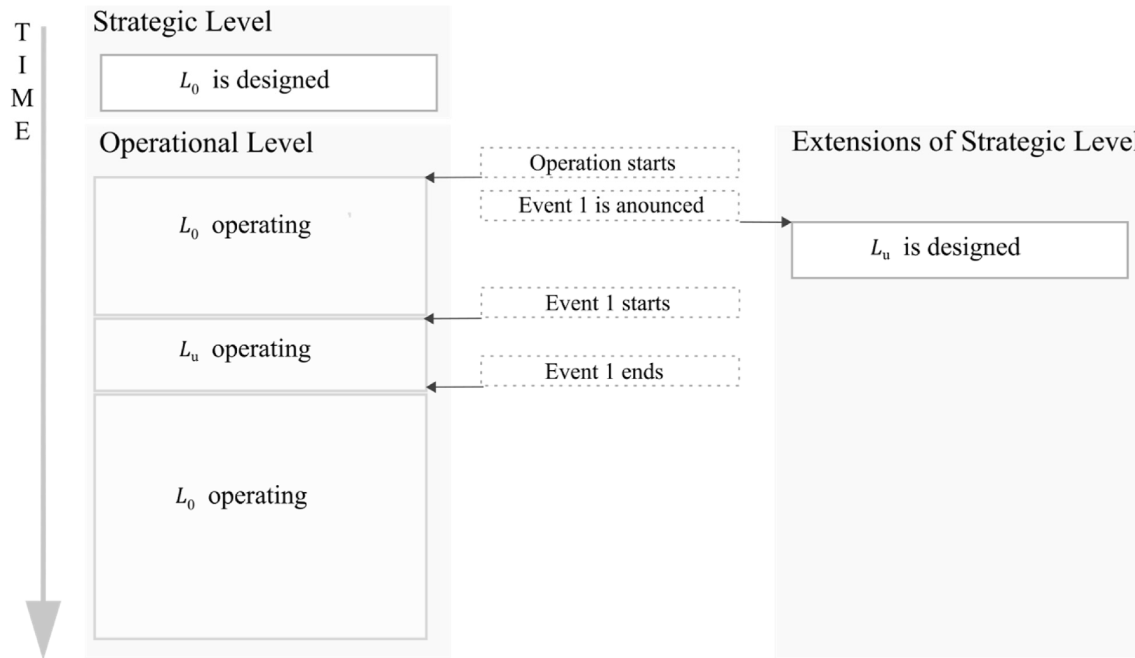

Fig. 1. Line Planning decisions for Flexible Line Plan.

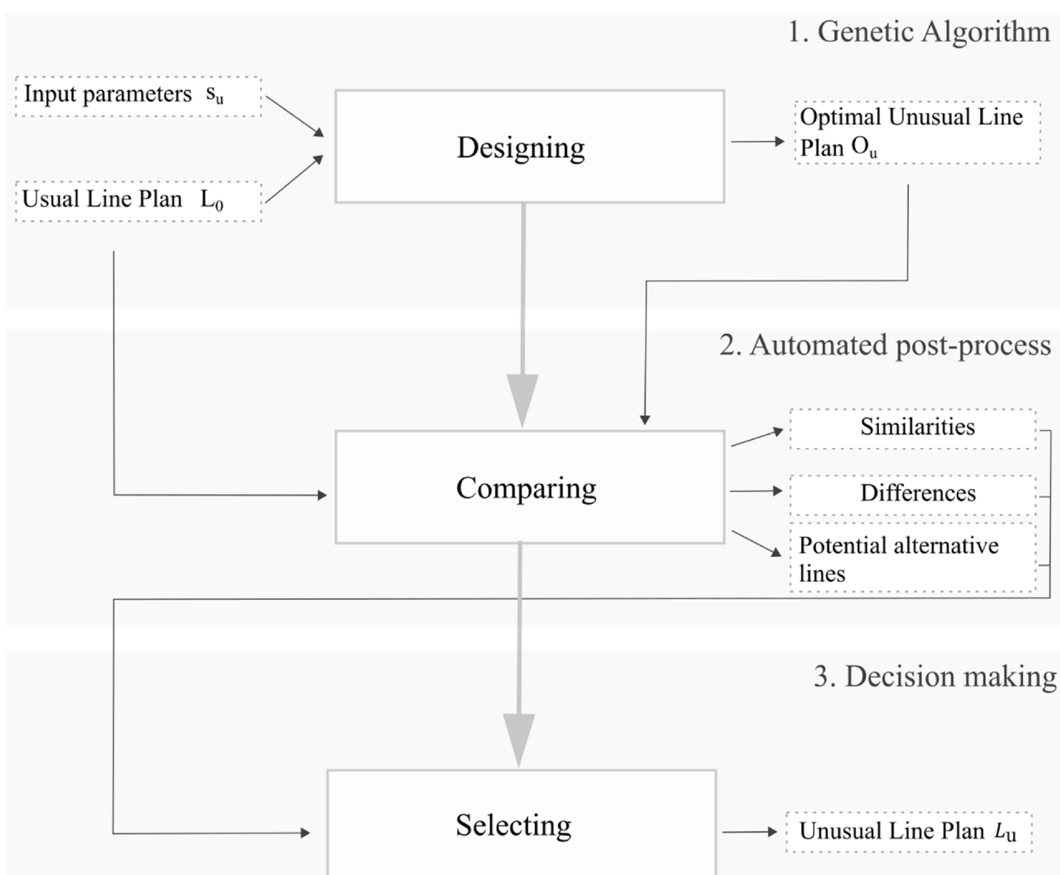

Fig. 2. ALH heuristic.

Table 2
Techniques to design alternative lines.

	Technique		
	ALH	GA-P	GA-F
Requires L_0	Yes	Yes	Yes
Requires O_u	Yes	No	No
Calculates total travel times as in GA	Yes	No	Yes
Defines the max number of alternative lines	Yes	No	Yes
Only considers alternative lines similar to L_0 lines	Yes	No	No

Table 3
Input parameters defining the usual situation in Mandl's network.

Parameter	Value
IN	Mandl's network
OD	The total demand is 15 570 travels. Nodes 1, 6 and 10 have the highest demand and the demand between nodes is symmetric.
LT	30 km/h
ML	50 min
$\overline{ST}$	1 min
FS	4 units
$\overline{TST}$	10 min

5. Proof of concept

Two networks are used to test the efficiency of a *Flexible Line Plan*. The first one is the Mandl's network, which models a small area for which a bus service is designed. It is assumed that an event has been announced and changes in the usual conditions of service are foreseen. Then, a larger network is used that models the infrastructure of the city of Cuenca in Ecuador. This city has a greater demand for the service and many more stops and bus lines. In this network, 2 massive events are also announced and for each of them alternative lines are designed that allow us to quantify the benefits of our methodology.

The required data for both networks and detailed information about all the results presented below are available here: https://www.mech.kuleuven.be/en/cib/lp/mainpage#section-22.

5.1. Flexible line plan in a small network

The first part of this proof of concept is to design alternative lines for the *Flexible Line Plan* on the well-known benchmark network described in 1980, by Mandl (Mandl, 1979). This small network has 15 nodes and 21 undirected links. Fig. 3 illustrates the network and the travel times on each arc.

The usual situation s_0 for Mandl's network is defined according to Table 3.

In this case, L_0 is designed using the GA and results in the 4 lines presented in Fig. 4. It copes with all demand in 171 755 min.

We use a fictitious event to illustrate an unusual situation: the situation s_u. This situation models congestion at the center of the network due to an event attracting twice the number of passengers to node 10. Links 8–10 and 10–11 are the most important connections to node 10, therefore they experience an 80% increase in their travel times, these links are called red links in Fig. 5. Likewise, other links are affected: those where the travel time is increased by 60% (orange links) and those with a 40% increase (yellow links). These progressive changes in speed simulate the expansion of real congestion. Fig. 5 presents the demand in node 10 and congestion for situation s_u.

The ALH described in Section 4.3, is now applied to design alternative lines for situation s_u. In the first step, the new travel times and demand are considered by GA to create the O_u for this specific unusual situation. This line plan is depicted in Fig. 6, the new lines give priority to node 10: now 4, instead of 3 lines, are serving that node.

The second step of ALH, produces the matrix E shown in Table 4 regarding the similarity of the lines. For instance, line 1 of O_u is more similar to lines 2 and 3 of L_0 than to lines 1 and 4. Also, the results exhibit that there are no identical lines between both line plans. Thus, although O_u is the best solution for the event, changing to that line plan means changing all the usual lines. This would imply a lot of discomfort for regular passengers and would make the operation complicated. Consequently, adapting the usual plan will contribute to finding a balance between making changes towards the " optimal solution" and staying with the usual line plan that was not designed for this situation.

All the lines of O_u contribute to save travel time during the event, therefore they are in set P presented in Table 5.

Moreover, the automated process identifies that 36.05% of the OD-pairs improve their travel times using O_u (Table 6). The set of potential alternative lines is built according to their contribution to saving travel time. At the top of this ranking is line 4 of O_u, followed by lines 1, 3, and then line 2.

According to the results, O_u spends more time in the congested zone but the use of the "less congested links" (yellow ones) is increased and the most congested ones are avoided as can be seen in Table 7. Although demand changes significantly from one situation to another, we note

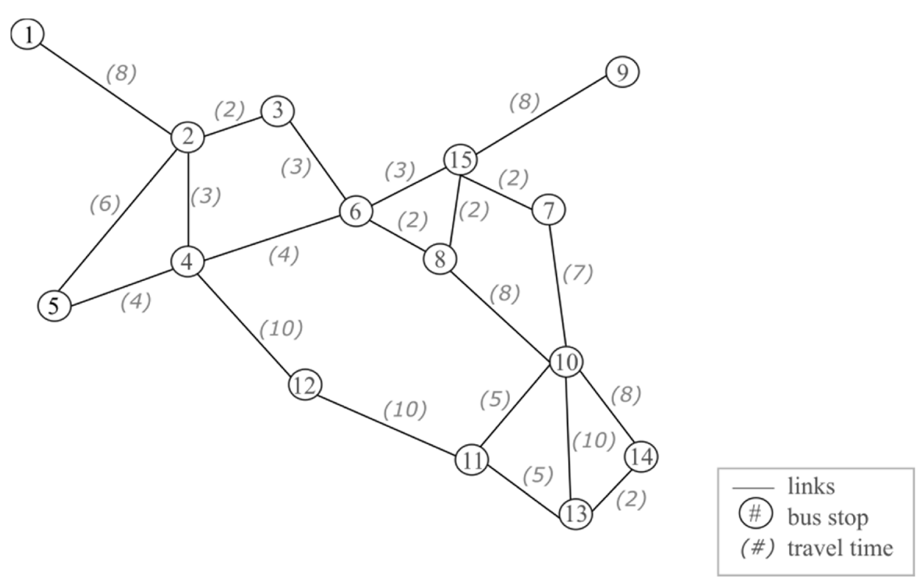

Fig. 3. Mandl's Network.

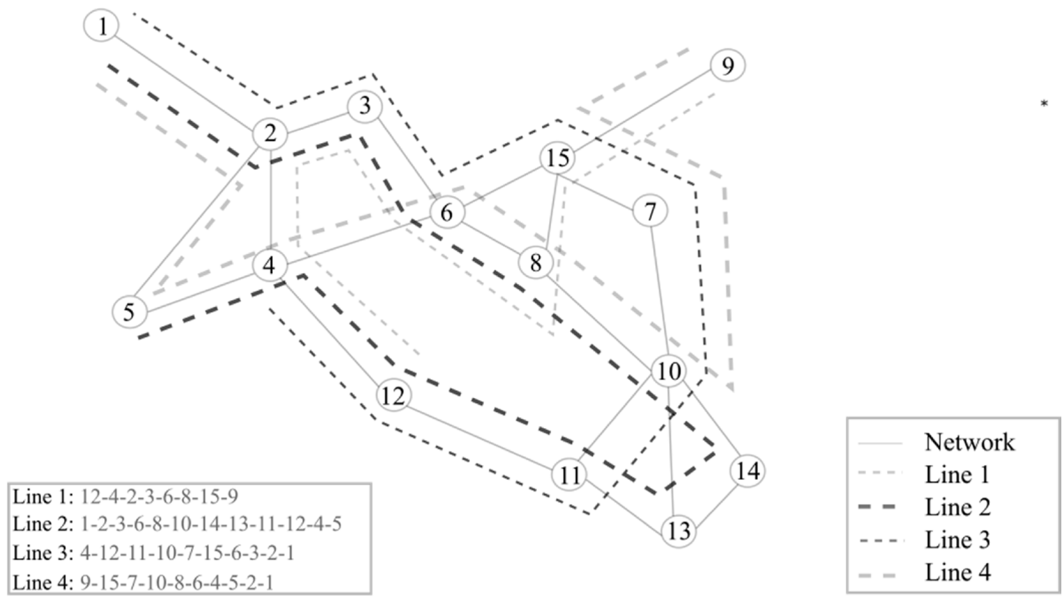

Fig. 4. L_0 of Mandl's network.

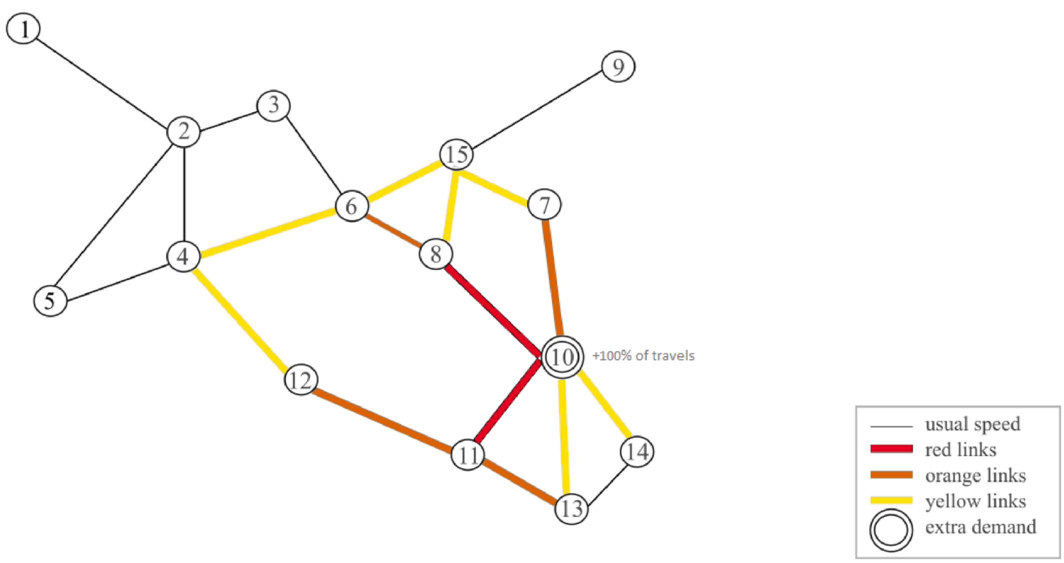

Fig. 5. Changes on demand and congestion in..s_u

that an optimal plan for the unusual situation uses less congested links, helping to reduce travel time and congestion. If the usual line plan keeps in operation, many more buses would be using the red and orange zones. Therefore O_u seems to be a better solution during the unusual situation.

The last step of ALH uses these results to produce the unusual line plan L_1. For situation s_u, ALH designs one alternative line to get significant savings in travel time, Fig. 7 shows the usual line 2 and the recently created alternative line 2a, which corresponds to line 4 of O_u. It is clear these lines are very similar; the only differences are a detour between nodes 8 and 10 and a new departure station in order to respect the maximal line length.

Table 8 presents the total travel time T of the line plans: L_0, L_1 and O_1, calculated for both situations s_0 and s_u. Rows labeled as ALH, GA-P and GA-F exhibit the performance of a L_1 designed by those techniques according to the algorithms described in section 4.3 and 4.4. In all cases, L_1 has one alternative line.

Line plan L_1, regardless of the technique used to design it, has a performance closer to the optimal in s_u, this is a promising performance that does not occur in s_0, as expected. L_0 always underperforms L_1 in the unusual situation as shown in Table 9. Smaller ratios are desirable.

The alternative lines should resemble the usual lines as much as possible. ALH is the technique that best meets this objective as shown in Table 10, where the percentages of bus stops shared between the usual and alternative lines are presented. The good and bad results of GA-P and GA-F respectively, are just a coincidence, since neither of these techniques ensures similarity. These results can be explained by the size

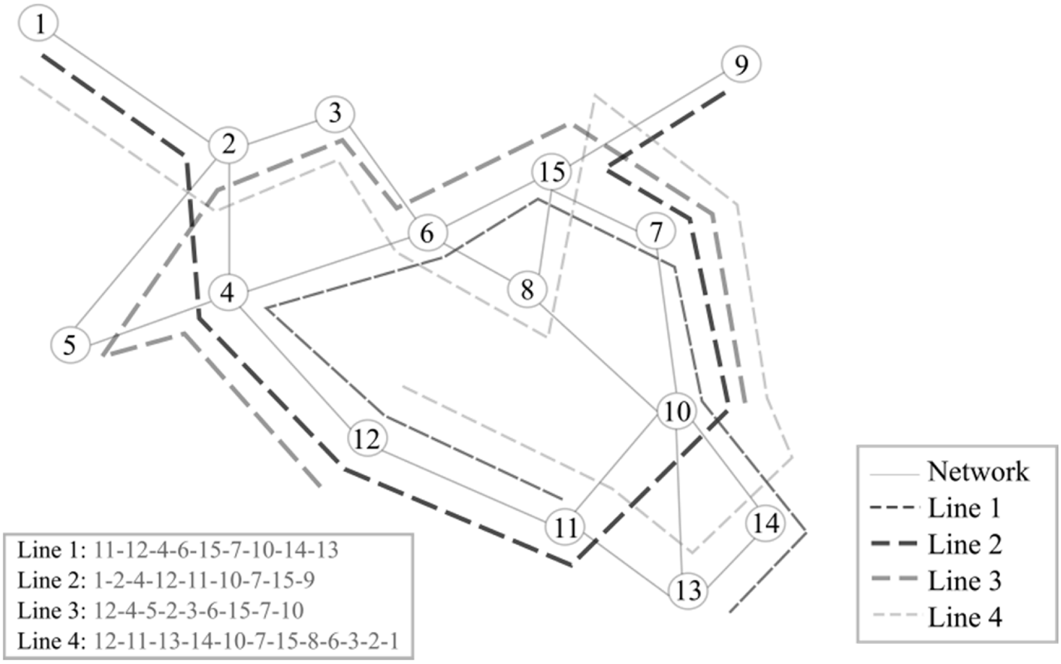

Fig. 6. Unusual Optimal line plan O_u of Mandl's network.

Table 4
Matrix E of similar lines between O_u and..L_0

L_0/O_u line	1	2	3	4
1	0–50%	51–70%	51–70%	0–50%
2	71–90%	51–70%	71–90%	71–90%
3	71–90%	71–90%	71–90%	71–90%
4	0–50%	71–90%	71–90%	51–70%

Table 5
Set P of Potential Alternative Lines.

Line	Travel Time savings (min)
4	20 138
1	16 216
3	4 088
2	2 036

Table 6
Changes on travel times when O_u is used instead of L_0 in..s_1

Type of change	% Of OD-pairs
Better travel time	36.05%
Worse travel time	22.09%
Unchanged travel time	41.82%

Table 7
Use of the congested zone.

Type of link	Travel Time (minutes) spent in the congested area by L_0	O_u
Red	293 325.00	277 125.00
Orange	109 894.64	98 735.36
Yellow	37 771.07	69 763.93
Total	**440 990.71**	**445 624.29**

of the network as it limits the diversity of feasible lines available. Later, when a real network is analyzed, it will be seen that neither of these two techniques designs similar lines.

Although GA-P and GA-F design line plans with better T than ALH in s_u, the latter is the technique that requires the least changes, therefore it is the most appropriate from the users' perspective. Thus the L_1, designed by ALH is the one selected in this *Flexible Line Plan F* for Mandl's network, although the result for GA-P is certainly also acceptable here.

5.2. Flexible line plan in a real-size network

The public transport in the Ecuadorian city of Cuenca is mainly provided by bus services. This service has a fleet of around 500 buses serving 58 lines. The largest line has a route length of 48.6 km and connects 24 zones within the city and the surrounding areas with an average frequency of 1 bus every 7 min. More than 1 000 bus stops are strategically located on an area of roughly 72 km^2; only a few of them serve as a terminal. Around 700 000 travels per day are made by the public transport system. The current line plan consists of 58 bus lines.

Now, a *Flexible Line Plan* for Cuenca, considering two unusual situations s_1 and s_2 is designed. Situation s_1 corresponds to an event in the stadium of Cuenca, leading to extra demand for the bus service and congestion in the area around it. The stadium of Cuenca is placed in a new area with offices, shopping centers and other services attracting a large number of people. Situation s_2 considers events in the colonial center of the city, where the city hall, shops and many tourist attractions are located. Therefore, this area is currently the one with the highest demand for travel. Moreover, the area is the venue of many traditional events attracting more demand and complicating the traffic flow on its narrow streets. Situation s_2 describes such an event. Fig. 8 shows the bus

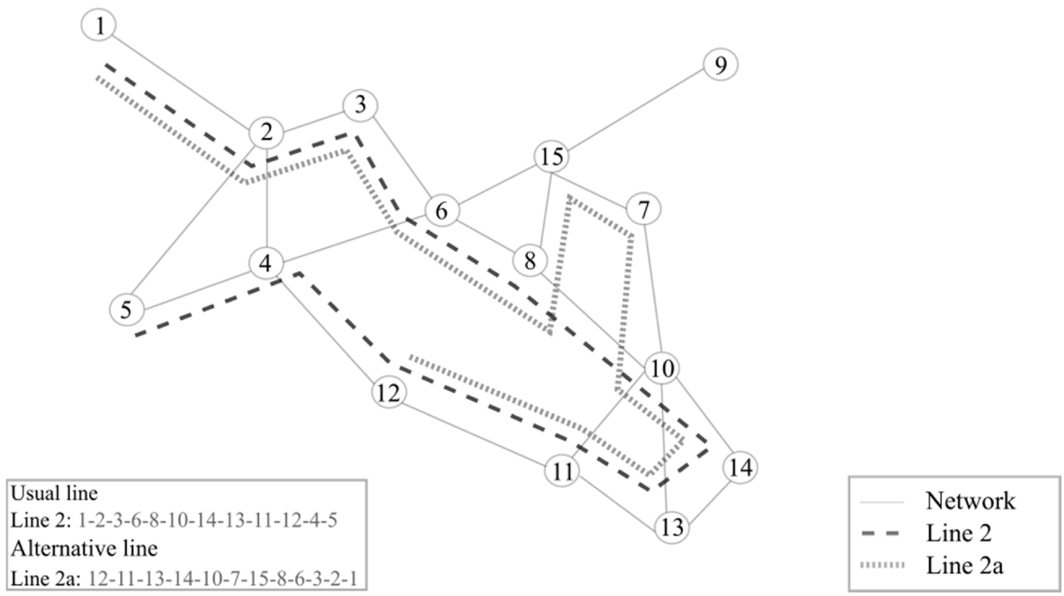

Fig. 7. Alternative Line of Mandl's network during s_1, designed by ALH.

Table 8
T in minutes of some line plans for Mandl's network.

Situation	s_0			s_u		
Algorithm/Line Plan	L_0	L_1	O_u	L_0	L_1	O_u
GA	171 755		185 535	569 497		532 982
ALH		173 540			549 244	
GA-P		182 635			545 470	
GA-F		174 565			541 111	

Table 9
Ratios between the line plans in usual and unusual situations.

	L_1/L_0		L_1/O_u
Situation	s_0	s_u	s_u
ALH	1.010393	0.964437	1.030511
GA-P	1.063346	0.957810	1.023430
GA-F	1.016361	0.950156	1.015252

Table 10
Similarities of the alternative lines designed by different techniques.

Line of … vs. line of L_0	% Of similarity
L_1 designed by ALH	83.33
L_1 designed by GA-F	63.64
L_1 designed by GA-P	81.82

stops and links of the infrastructure of Cuenca. The locations of the colonial center and the stadium are indicated.

Table 11 presents the parameters of the usual situation as well as the changes that they experience in each of the unusual situations.

An essential point to consider is that in both unusual situations, more people will use public transport. These are indicated as additional travels in Table 11. Also, the speed on certain links decreases. As a result, it makes no sense to compare the total travel time T between different situations. It only makes sense to compare different line plans for each situation separately.

Furthermore, the current line plan in Cuenca have emerged over time to meet new demands; therefore, they are not the product of a formal design process or optimization. Therefore, in this study we use our GA to design the usual line plan for Cuenca. In this way, all the line plans considered in this section, will have been designed with the same objective function, and can be compared fairly.

5.2.1. *Stadium situation*

First, the unusual stadium situation is considered. The GA designs the line plans L_0 and O_1 and later, the three methods: ALH, GA-P and GA-F adapt the usual line plan to situation s_1. Fig. 9 presents the total travel time T of these line plans calculated for both usual and unusual situation. L_0 and O_1 perform the best for their respective situation and any of the unusual line plans L_1 always improves the T with respect to L_0 in situation s_1. ALH designs the second-best L_1, and is slightly outperformed by the one designed by GA-P. The GA-F method produced the worst adaptation of L_0.

O_1 reduces the T by 60 872.98 min from the T using L_0 during situation s_1. Thus, assuming that O_1 is the optimal line plan for that situation, this value is the upper bound of the time savings that the adapted line plans will be able to achieve.

Table 12 presents the number of identical lines compared to L_0. It is clear that ALH and GA-F produce line plans that most resemble L_0. Only four lines are different. GA-P, on the contrary, has 15 different lines. Although GA-P results in a slightly better T, the difference with the line plan from the usual situation is needlessly high. Therefore, we conclude that the line plan generated by ALH is the best compromise between performing well during the unusual situation and not disturbing the regular performance too much.

The efficiency of ALH to adapt line plans through few but well-selected changes was already seen in Section 5.1 and is now confirmed for this unusual situation in Cuenca network. Since passengers will appreciate improvements in performance during an event by only changing a few lines, the line plan designed by ALH is preferred for this unusual situation in Cuenca. ALH saves around 50% of the travel time that can be saved according to the values achieved by O_1. Moreover, ALH not only limits the number of alternative lines to four, but also

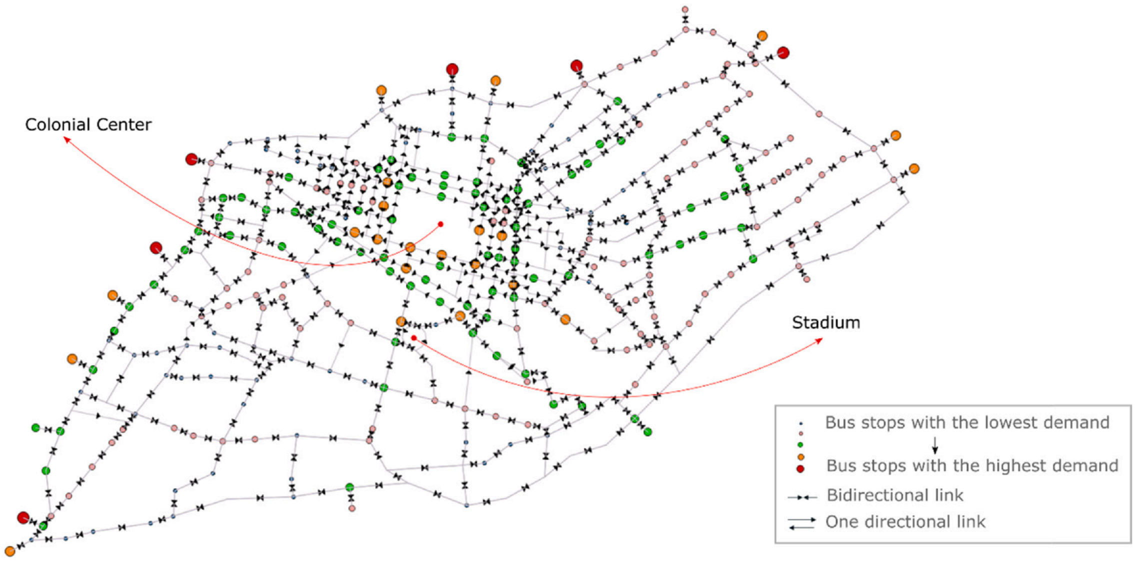

Fig. 8. Infrastructure available for public transport in Cuenca.

Table 11
Usual and unusual situations of Cuenca.

Parameter	Usual Situation	Changes on unusual situation	
	s_0	s_1	s_2
IN	Cuenca's network: 422 nodes and 482 links		
od	700 037 travels/day	+7 584 travels to and from the stadium.	+13 703 travels to and from the colonial center.
lt	15, 40 and 30 km/h: in the center, border and the rest of links respectively.	−30% to −90% in the speed of the links affected by the event.	−30% to −90% in the speed of the links affected by the event.
ml	147.3 min		
$\overline{st}$	0.5 min		
fs	58 units		
$\overline{tst}$	10 min		

Table 12
Similar lines among solutions.

		% Of common bus stops			
L_1 designed by… vs L_0	# Identical Lines	0–50	51–70	71–90	91–100
ALH	54	3	1	0	0
GA-F	54	4	0	0	0
GA-P	43	10	3	2	0

ensures that the new routes are somewhat similar to the usual ones, this can be seen in Fig. 10.

In order to illustrate the typology and consequences of the changes suggested by ALH, we discuss two modifications in more detail. The event in the stadium attracts people from an area of urban growth, indicated as the origin at the bottom of Fig. 11. Under normal conditions, travelling from that area requires two transfers. With the alternative line a direct travel is offered which significantly reduces the travel. Moreover, this is a clear example of how our approach designs an

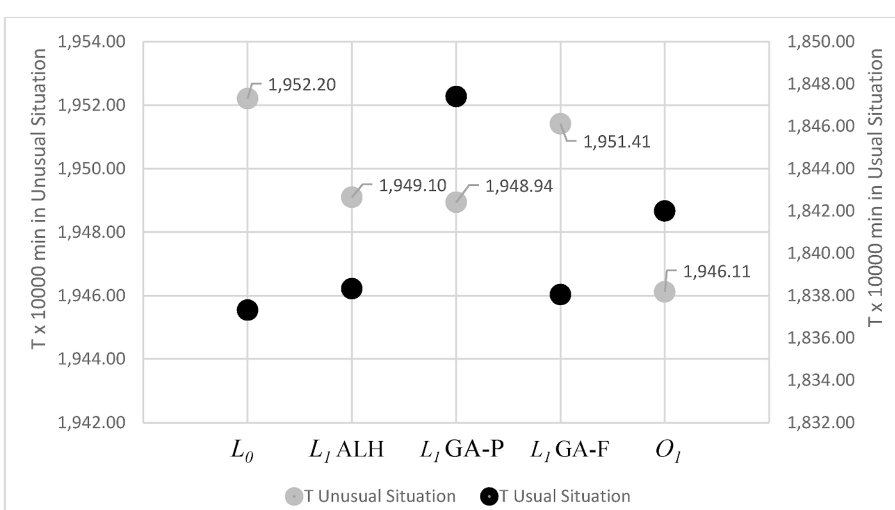

Fig. 9. Total travel times of different line plans in situations s_0 and..s_1

Fig. 10. Alternative lines for L_{u1}

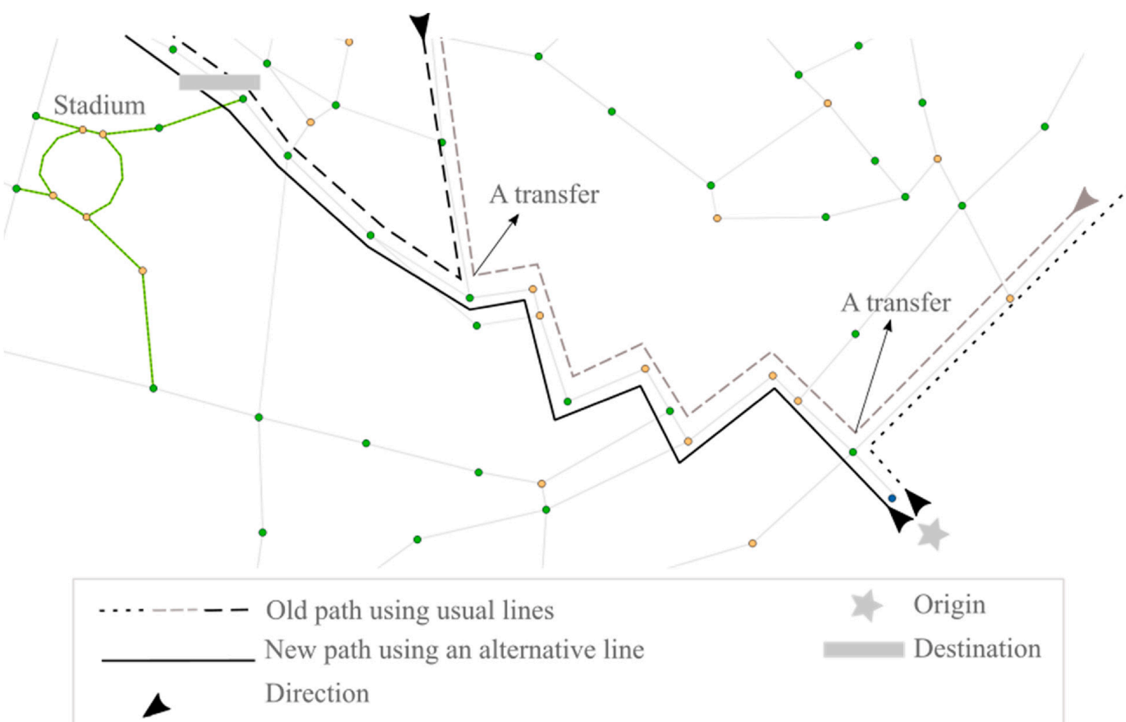

Fig. 11. An improved travel to the stadiuḿs zone.

alternative line, that instead of avoiding the area affected by the event, uses it to better serve the additional demand.

The following example deals with the unusual situation by avoiding the congested area. The passengers illustrated in Fig. 12, usually travel through the stadium area, but this path is heavily congested when the event takes place. ALH designs a line with a new path moving around this area and reducing travel time.

Table 13 presents the use of the congested zone for both line plans. The routes of alternative lines of L_1 reduce the time spent when travelling around this area.

When using ALH, it is observed that 90.28% of passengers preserve their usual travel times, while 5.34% reduce their time and 4.38%

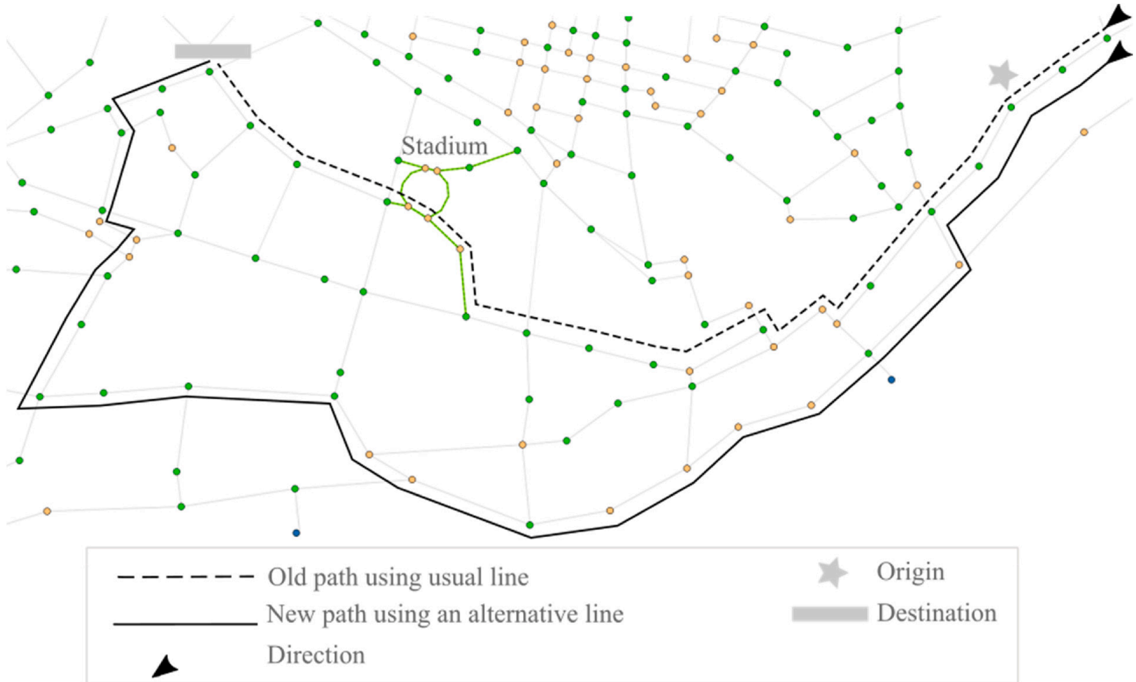

Fig. 12. A line avoiding the congested zone.

Table 13
Use of the congested zone in situation..s_1

Type of link	Travel Time (minutes) spent in congested area by L_0	L_1
Red	58 048.84	58 093.21
Orange	171 742.48	168 469.94
Yellow	617 626.77	598 884.65
Total	**847 418.09**	**825 447.80**

increased it. As expected, the alternative lines address the unusual situation through the following strategies: (1) improving the travel time to the event, (2) rerouting lines that pass through the event area with a lot of passengers having their destinations elsewhere, (3) keeping the lines for the rest of the passengers unchanged as much as possible.

5.2.2. Colonial center situation

Next situation s_2 considers events in the colonial center. Table 11 showed the changes on demand and speed related to that situation. During the event, the congestion decreases proportionally to the distance from the event. This is illustrated in Fig. 13. The area attracting most of the people is illustrated, together with how the links around it exhibit different levels of congestion.

First O_2 is designed and the comparison starts. Then, the unusual line plan L_2 for situation s_2 is designed running the heuristic ALH, GA-P and GA-F. The line plan designed by GA-P has 14 different lines compared to O_2. The value of the *numAlternativeLines* of ALH is 28, which corresponds to around half of the lines that could be changed. However, when running ALH, a solution is designed that modifies only three lines and that solution achieves the best results. The number of fixed lines when running GA-F was set to 3 that is the number of alternative lines of the line plan design by ALH.

Also, the *ALH* algorithm changes the lines of the usual plan for those that produce the most savings in travel time in O_2, but not every change keeps the line plan feasible, and every change must be made with a line that is somewhat similar to the alternative line. These constraints contribute to designing a solution with only a few modified lines. Again, the designed solution looks for a compromise between generating the least hassle for the regular users and the most benefits for the unusual situation.

The performance of the different solutions is presented in Fig. 14, where again it is noted that the unusual plan L_2 ALH is clearly a more efficient solution than the usual plan for the event situation, requiring only a few changes. Indeed, ALH obtains around 60% of the travel time that could be saved. In this case, L_2ALH also clearly outperforms L_2GA-P and L_2GA-F in the unusual situation, with the same or a smaller number of changing lines.

During the event in the colonial center, many usual paths change into others that prefer congestion-free links. Table 14 shows the time spent in red, orange and yellow links by both line plans.

ALH keeps the usual travel time for 91.40% of passengers, while 4.51% reduce their travel times and only 4.09% of passengers are affected by the increase in travel times.

5.3. Computation time

All the experiments in this study were performed using an Intel Core i7 desktop PC, with 12 GB of RAM. The CPU time required to get a solution for Mandl's network is around one minute, while for Cuenca, due to its size, it takes on average six hours for a single run. In order to eliminate the randomness effect of the algorithm on the solutions, the GA always ran several times for the same situation. From these runs, the line plan with the best performance is selected. In the case of Mandl, the

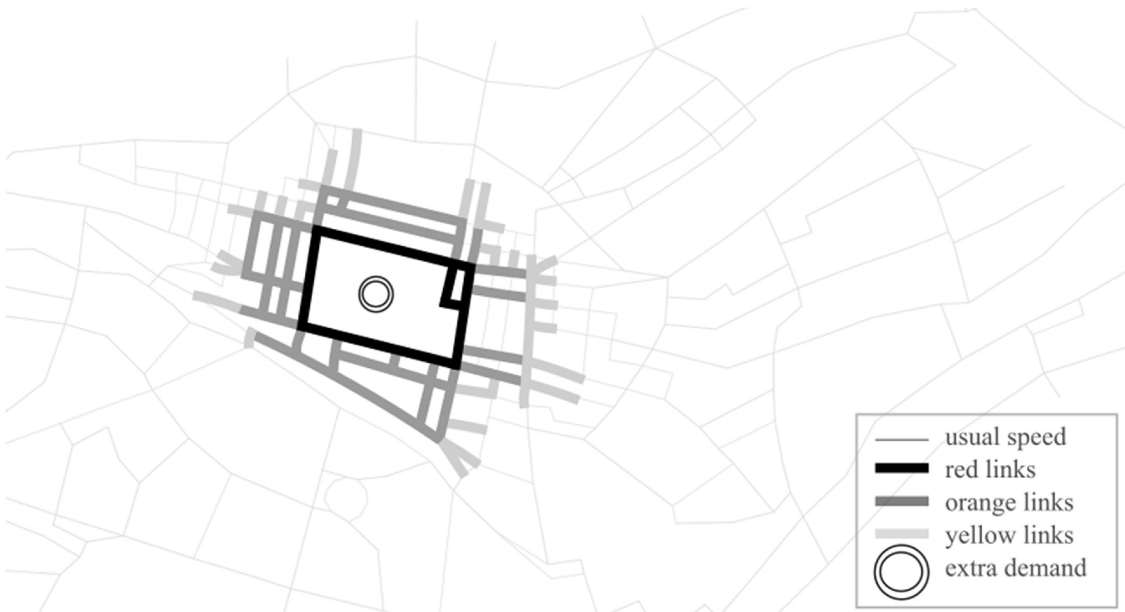

Fig. 13. Congested zone during an event in the downtown.

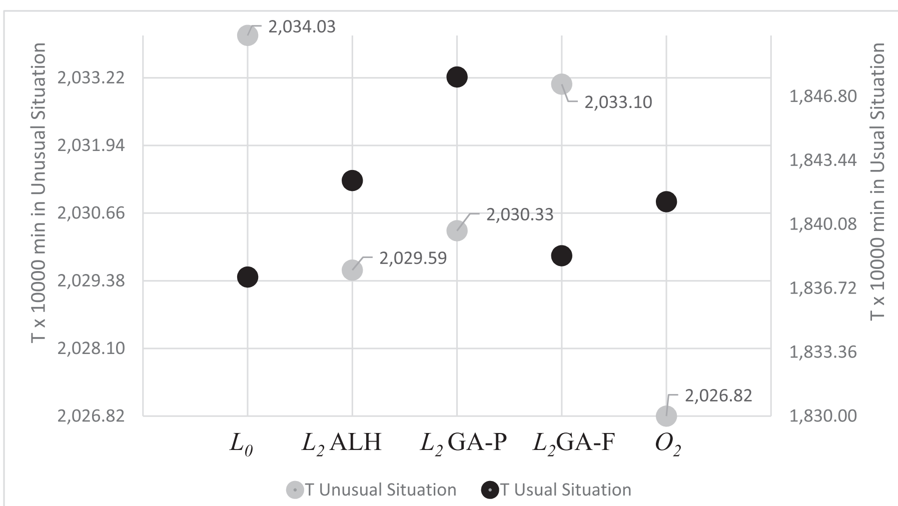

Fig. 14. Total travel times of different line plans in situations..s_2 *and* s_0

Table 14
Use of the congested zone in situation..s_2

Type of link	Travel Time (minutes) spent in congested area by L_0	L_2
Red	195 693	182 558
Orange	578 554	566 252
Yellow	205 727	200 118
Total	**979 974**	**948 928**

algorithm runs 10 times, but for Cuenca, 3 runs have been performed. For the Cuenca cases, 80 individuals are considered in the population and 400 iterations. For Mandl, 40 individuals and 100 iterations are performed.

6. Conclusions

In this paper, the concept of a *Flexible Line Plan* is introduced. This focuses on reducing the negative effects on the performance of a service caused by changes in demand and congestion related to unusual events such as markets or sports events. The concept proposes to change a limited number of bus lines during these unusual events. This should allow to significantly adapt the line plan to the new situation, but at the same time keep the current service the same for most regular passengers. The ALH heuristic was created to design these unusual line plans.

The heuristic is applied on two networks and was compared with other techniques. The results showed that ALH can identify the usual lines that should be modified and design a set of alternative lines to meet the changes imposed by the unusual situation.

The time saved by the *Flexible Line Plan*, with alternative lines reaches, 50–60% of the maximum time that could be saved. It results in a reduction of bus lines using the congested area and not only makes the system more attractive for the current users, but it also serves the additional passengers attracted by the event.

Further research could be devoted to designing an automated system to inform the passengers about the changes, in order to facilitate the implementation of a *Flexible Line Plan*. Some datamining techniques could be useful to evaluate the similarity of the alternative lines and define patterns, this could decrease the time required to create a specific solution for each unusual situation. The extra demand and congestion on the unusual situations could be designed based on historical data to mimic conditions closer to real disturbances. Finally, the concept of *Flexible Line Plan* could also be useful for emergency situations in a part of the city, if the calculation times can be further reduced.

CRediT authorship contribution statement

Elina Avila-Ordóñez: Conceptualization, Methodology, Software, Data curation, Writing – original draft, Investigation, Visualization. **Chris M.J. Tampère:** Investigation, Validation. **Pablo Vanegas Peralta:** Writing – review & editing. **Pieter Vansteenwegen:** Conceptualization, Methodology, Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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