A Multiple Criteria Heuristic Solution Method for Locating Near to Optimal Contiguous and Compact Sites in Raster Maps

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Abstract. A high performance heuristic solution method is proposed able to locate near to optimal sites composed by a given number of cells (raster structure). These sites must be compact and maximize levels of the sites intrinsic multiple criteria suitability. To validate the heuristic approach, a comparison with a mathematical formulation is performed with afforestation data of regions within the Netherlands, Denmark, and Flanders. This reveals that the heuristic is considerably faster than the mathematical method and the objective values obtained with the two approaches are substantially similar. A sensitivity analysis shows that the region's homogeneity plays an important role in the performance of the process identifying most favourable sites. Moreover, computation time follows a power model in the number of cells forming the site.

Keywords: Site Location, Heuristic, Exact Methods.

1 Introduction

In the practice of environmental conservation and land use planning sites are searched for fulfilling shape (e.g. contiguity, compactness) and size requirements, satisfying also particular attribute criteria. This paper presents a mathematical formulation and a heuristic solution method for identifying compact and contiguous sites formed by a set of cells maximizing the intrinsic multiple criteria suitability of each site. The notion of compactness is associated with firmly packed sites. In this paper we adhere to *the earliest attempts to develop a compactness index based on perimeter to area ratios* (Maceachren, 1985). On the other hand, a

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site is contiguous if one can walk from an identified parcel to another without leaving the site (Xiao, 2006). Therefore, compactness implies contiguity, but not the opposite.

Contiguity and compactness requirements and intrinsic multiple criteria suitability of the cells are all involved in finding optimal sites. Thus, Multiple Criteria Decision Analysis (MCDA) is included as part of the proposed mathematical and heuristic approaches for site location.

Although *mathematical optimization methods have been used for about 30 years in areas like forest planning* (Williams and ReVelle, 1997), the size of the problems that can be handled in a practical way remains limited. Nevertheless, these methods can be very useful as part of an adaptative, learning process of problems at hand (Hof and Bevers, 2000). Therefore, the proposed heuristic for site location is evaluated taking the mathematical formulation results as a reference. Even though the proposed mathematical formulation by itself does not ensure contiguity, it makes use of the benefit gained through the boundaries of contiguous pixels, tending to minimize the perimeter while the area is constant. A weight configuration giving the same importance to each criterion fulfills the contiguity and compactness requirements.

This paper is organized as follows: Section 2 reviews techniques applied in site location problems; Section 3 describes the data and approaches used in this study,; Section 4 analyzes the results and Section 5 summarizes the conclusions.

2 Literature Review

2.1 Exact Methods for Site Location Problems

Exact methods include enumeration and mathematical programming, as well as many specialized algorithms that have been developed for particular optimization problems (Williams and ReVelle, 1997).

2.1.1 Mathematical Programming Methods

Several approaches for site location problems make use of Linear Programming (LP) models where the variables are integers (Integer Programming - IP). LP/IP attempts to maximize (or minimize) a linear function, constraining the values of the decision variables. Hof and Bevers (2000) formulated four linear programming examples including constraints to avoid adjacency in order to account for biological dispersal. The constraints relate population in a habitat area in a time period (t) to the populations in other areas in a previous time period ($t-1$), while taking into account the population growth and the immigration dispersion. Church and ReVelle (1974) introduce the Maximal Covering Location Model (MCLM), which minimizes the number of facilities to cover each and every demand point on a network. MCLM is modified (Dimopoulou and Giannoikos, 2001) to determine the optimal deployment of available fire-fighting vehicles. In order to reduce the vulnerability of elements like species, communities, or endemic plants, Church et al, 1996 develop a mathematical model for selecting sites for diversity conservation (Biodiversity Management Areas - BMAs). Since the solutions are composed of isolated planning units, to avoid fragmentation, Fischer and Church, 2003 formulate a mathematical model including the objective of minimizing the outside perimeter of the selected areas. The idea to represent a mosaic of n cells as a planar graph with vertices and edges is presented by Williams (2001; 2002). Each cell is equated with a vertex, and each adjacency relation between a pair of cells is equated with an edge. Shirabe, 2004 also applies this idea to formulate the necessary and sufficient conditions for assembling a connected region with a desired degree of perforation, from no hole to a largest possible number of holes.

2.1.2 Enumeration Methods

Enumeration methods evaluate all candidate solutions (explicit enumeration - brute force), or identify a set of efficient solutions (implicit enumeration), and select the one that optimizes specific criteria. Since the computational cost of this sort of search is proportional to the number of candidate solutions, it is typically used in problems of limited size. Gilbert et al (1985) develop an interactive multi-objective algorithm for allocating an area of land (set of cells). This approach partially generates a set of efficient solutions that achieve four objectives: minimize development cost, minimize the distance to desirable cells (amenity distance), maximize distance to undesired cells (detractor distance) and minimize a shape objective expressing contiguity. Similarly, Diamond and Wright (1991), apply an implicit enumeration method, which is based on irregular grids to generate multi-objective contiguous sites. The tracts of sites maximize the level of suitability regarding cost, area, and shape.

2.2 Approximate Methods for Site Location Problems

Heuristic programming techniques have been developed to solve problems of which the solutions each constitute just one point or a small set of points in a very large, and possibly infinite space, the search space (Siklossy and Marinov, 1971). A heuristic is a problem-specific way of directing problem solving. *It seeks to obtain good, that is, near-optimal solutions at relatively low computational cost without being able to guarantee the optimality of solutions* (Dorigo, 2004).

2.2.1 Heuristic Approaches

To deal with the problem of generating contiguous and compact districts while providing population equality and retaining jurisdictional boundaries, Mehrotra et al (1998) develop an optimization-based heuristic capable of considering many potential districts. The problem is represented with a graph, where each node is associated with the population of a county (unit), and an edge exists when two geographical units are neighbours. A penalty cost is assigned to every potential district, measuring its *deviation* from an ideally compact district. To explicitly manage the shape in site location problems, Brookes (2001b) proposes the Parameterized Region-Growing (PRG) process, which starts from a seed cell. This algorithm is a fusion of two ideas: Simple Region-Growing (SRG) and Parameterized Shape-Growing (PSG). The SRG algorithm iteratively adds the most suitable neighboring cell. If two or more cells have equal suitability then the one closest to the seed is chosen. The PSG algorithm uses the same incremental process as the SRG but with a shape-suitability score determined by the distance and direction of the cell to the seed. PRG combines PSG and SRG through a weighted average of the two scores. The suitability of a cell S with shape score *Ss* and underlying cell suitability *Sc*, when the trade-off is T, is given by $S=[T*Ss]+[(1-T)*Sc]$ (Brookes, 1997). PRG generates promising regions with a specific shape when an operator chooses the approximate location, shape and orientation of the regions. Nevertheless, an appropriate parameter setting is required. Church et al (2003) develop a Patch Growing Process (PGP) to generate feasible patches with reference to a seed cell. Once the seed patch is defined, the neighbors to the patch are placed on a list in a random order. Each cell in the list is analyzed in terms of the number of edges (e) that it shares with the current patch (from 1 to 4). The composite suitability of the i_{th} cell (CS_i) is defined by $CS_i = Suit_i + N.e_i$, where $Suit_i$ is the suitability value of the cell itself, N the weight attached to the number of edges shared with the existing patch, and e_i the number of edges that the i_{th} cell shares with the current growing patch. Then the list of neighboring cells is ordered according to the composite suitability, and the top X percent of the cells on this list are added to the patch.

2.2.2 Metaheuristic Approaches

When the heuristics are general-purpose methods that can assist in finding high quality solutions, those are called metaheuristics. Genetic algorithms and simulated annealing are frequently applied metaheuristics in site location problems.

Stewart et al (2005) propose the integration of Genetic Algorithms (GA) with a formulation for Reference Point Goal Programming (RPGP). The general formulation of the objective function for RPGP considers in the first part simple additive attributes which associate costs or benefits with the allocation of any particular land use to a specific cell. In the second part it considers spatial attributes indicating the extent to which the different land uses are connected or fragmented across the region. The PRG approach proposed by Brookes (2001b), described earlier in this paper, is combined with a Genetic Algorithm for Patch Design (GAPD) (Brookes, 2001a) in order to explicitly handle dynamic and static criteria. Additional approaches applying Genetic Algorithms for the site location problem are proposed by Li and Yeh (2004), to deal with multiple criteria evaluation; and by Xiao (2006) to search locations represented under vector structures.

Other metaheuristics have also been applied in site location, Aerts and Heuvelink (2002) apply simulated annealing to allocate $N \times M$ pixels with K different types of land use. McDonnell et al (2002) compare greedy search and simulated annealing methods to construct spatially cohesive reserve areas. While the simulated annealing approach is similar to the one by Aerts and Heuvelink, 2002, the greedy search algorithm updates an existing solution by adding one unreserved site.

2.3 Multiple Criteria Decision Analysis

The term 'Multiple Criteria Decision Analysis (MCDA)' is used by Belton and Stewart (2002) as an umbrella term to describe a collection of formal approaches that seek to take explicit account of multiple criteria in helping individuals or groups exploring decisions that matter. Many site location problems involve also criteria coming from different actors, and selected sites must comply with more than one objective. To deal with these issues, several approaches for site search include Multiple Criteria Decision Analysis (MCDA); *where a criterion is a generic term including the concepts of attribute and objective* (Malczewski, 1999). While attributes are the properties of elements of a real world, an objective is a statement about the desired state of a system under consideration (Malczewski, 1999). Xiao (2006) states that when objectives conflict, it is often impossible to find a single optimum that dominates all other solutions. One of the dichotomies present in MCDA is Multiple Attribute Decision Making (MADM) versus Multiple Objective Decision Making (MODM). While MADM obtains levels of attributes through preferences in the form of objective functions and attribute weights, in MODM these levels are derived from the preferences among objectives and from the functions relating attributes to objectives (Malczewski, 1999). Several approaches for site location make use of MCDA. Li and Yeh (2004) apply MADM while Gilbert et al (1985), Stewart et al (2005) and Xiao (2006) apply MODM.

3 Materials and Methods

First part describes the process to prepare the data in order to make it usable by the Integer Programming (IP) as well as for the Heuristic formulation. Next, both approaches are described. Since the problem at hand considers levels of importance (weights) for the criteria involved in the location of optimal sites for afforestation of agricultural land, a MADM approach is applied in the current work. However, since the paper is dealing with general formulations for site location, it does not evaluate alternatives for weights computation. The weights are in fact considered as mathematical parameters from the optimization point of view. This condition does not restrict the applicability and generality of the proposed approach.

3.1 Data Preparation

Both, the IP formulation and the HMSL are tested with data (criteria) generated and validated in the AFFOREST project (EU 5th Framework Programme for Research and Technological Development), pertaining to the Netherlands, Denmark, and the region of Flanders in the North of Belgium. In this sense, a validation of the input data is out of the scope of the present contribution. Moreover, it is believed that the proposed approaches for site location can be applied on grids of cells with attribute data generated by any other application field.

In AFFOREST, the Environmental Performance (EP) is defined (Gilliams et al, 2005) as the combination of three Environmental Impact Categories (EIC): *total carbon sequestration* (composed by carbon sequestration in biomass and in soil), *nitrate leaching*, and *ground water recharge*. The EP for each land unit, represented by a class of cells *or pixels*, as a function of *time* after afforestation, is computed by means of a metamodel (details of the Afforest project can be found in Heil et al, 2007). While a *pixel class* represents all the cells that have the same characteristics (initial agricultural land use, soil type, annual average of precipitation amount, annual average of nitrogen deposition), *time lag* is a period (year) after afforestation. In this sense, *afforestation* is defined as the transformation of agricultural land into forest.

Environmental Performance (EP) criteria are applied to test the mathematical and heuristic approaches for site location. Each criterion is represented through a raster map upholding information of one EIC (carbon sequestration, nitrate leaching or ground water recharge), at a specific year after afforestation, and for a specific afforestation strategy (e.g. total carbon sequestration after 10 years of afforestation with oak under medium stand preparation and medium stand tending levels).

For the purpose of this study, each criterion (raster map) is first *normalized* (NC) in order to avoid the influence of the differences in value ranges; each pixel upholds a value between 0 and 1 (1 = the best). Since the Environmental Performance (EP) can be defined by one or more EICs, a multi-criteria objective function is required in order to find a set of pixels maximizing the involved EICs. It is essential to consider that *maximization of EP* implies *maximization* of carbon sequestration (CS) and ground water recharge (GWR), but *minimization* of nitrate leaching (NL) levels. Accordingly, two normalization functions are defined: equation 1 is applied when the highest EP is obtained as a maximization, and equation 2 when the highest EP is obtained as a minimization. In these equations, u refers to a specific EIC layer. While $f(u)$ is the pixel value in the u layer, γ_u and σ_u are respectively the maximum and minimum value within the u layer (i.e. within each criteria).

$$M_{\max} = \frac{f(u) - \sigma_u}{\gamma_u - \sigma_u} \tag{1}$$

$$M_{\min} = 1 - \frac{f(u) - \sigma_u}{\gamma_u - \sigma_u} \tag{2}$$

3.2 Integer Programming (IP) Formulation for Multiple-Criteria Site Location

In a first stage, a 0-1 linear programming formulation is developed for acquiring optimal sites composed by a set of pixels forming a compact and contiguous site with maximal performance according to one or more weighted *Normalized*

Criteria (NC). Compactness is achieved by increasing the number of boundaries that a selected cell shares with other cells that are also selected as part of the solution. This formulation implies the reduction of the patch perimeter, while the area is constant.

In the first part of the objective function (equation 3), $s_{i,j}$ is a binary variable upholding a value 1 if the cell in row i and column j is part of the target site. While P is the number of criteria considered in the decision problem, w_k is the importance weight for the k_{th} criterion, and $c_{k,i,j}$ upholds the value for the k_{th} NEIC layer in the i,j cell.

The second part of the objective function deals with the compactness criterion, where w_N is the importance weight assigned to this requirement. To construct a compact site, the model considers the Von Neumann neighborhood of a cell (upper, down, left and right cells). The binary variable $u_{i,j}$, will uphold a value 1 if the i,j cell is selected as part of the target site ($s_{i,j}=1$), and at the same time the upper neighbor cell is *also* selected. The same judgment is applied for assigning values to the binary variables $d_{i,j}$, $l_{i,j}$, $r_{i,j}$, which are associated to the *d*own, *l*eft, and *r*ight neighbor cell respectively. Since the objective function is dealing with normalized values, and applies the Von Neumann neighborhood, the sum of $u_{i,j}$, $d_{i,j}$, $l_{i,j}$, and $r_{i,j}$ is multiplied by a 0.25 factor. In this manner, while the maximum level regarding the compactness contribution of the i,j cell is 1 (four neighbors of $s_{i,j}$ are *also* selected), the minimum level is 0 (no neighbors of $s_{i,j}$ are *also* selected as part of the solution).

maximize:

$$\sum_{k}^{P}(w_k * \sum_{i}^{m}\sum_{j}^{n} s_{i,j} * c_{k,i,j}) * w_N * \sum_{i}^{m}\sum_{j}^{n} 0.25 * (u_{i,j} + d_{i,j} + l_{i,j} + r_{i,j}) \qquad (3)$$

Subject to:

$$s_{i,j} - x_{i,j} \geq 0 \quad \forall_{i,j}; \quad \forall x_{i,j} \text{ where } x_{i,j} \in \{u_{i,j}, d_{i,j}, l_{i,j}, r_{i,j}\} \qquad (4)$$

$$s_{i-1,j} - u_{i,j} \geq 0 \quad \forall_{i,j} \qquad (5)$$

$$s_{i+1,j} - d_{i,j} \geq 0 \quad \forall_{i,j} \qquad (6)$$

$$s_{i,j-1} - l_{i,j} \geq 0 \quad \forall_{i,j} \qquad (7)$$

$$s_{i,j+1} - r_{i,j} \geq 0 \quad \forall_{i,j} \qquad (8)$$

$$m_{i,j} - x_{i,j} \geq 0 \quad \forall_{i,j}; \quad \forall x_{i,j} \text{ where } x_{i,j} \in \{u_{i,j}, d_{i,j}, l_{i,j}, r_{i,j}\} \qquad (9)$$

$$\sum_{i}^{m}\sum_{j}^{n} s_{i,j} = M \qquad (10)$$

$$s_{i,j} \in \{0,1\} \tag{11}$$

$$u_{i,j} \in \{0,1\} \tag{12}$$

$$d_{i,j} \in \{0,1\} \tag{13}$$

$$l_{i,j} \in \{0,1\} \tag{14}$$

$$r_{i,j} \in \{0,1\} \tag{15}$$

The first set of constraints (equation 4) guarantee that $u_{i,j}$, $d_{i,j}$, $l_{i,j}$, or $r_{i,j}$ can have a value 1 if and only if the *i,j* cell is *also* selected ($s_{i,j}=1$). Constraint 5 assures that if $u_{i,j}$ is equal to 1, the matching cell in the set of binary variables *s*, will be necessarily equal to 1 ($u_{i,j} = s_{i-1,j}$). Constraints 6 to 8 articulate the previous rule for variables $d_{i,j}$, $l_{i,j}$, and $r_{i,j}$. The integrality constraints in equations 5 to 8 tend to effectively reduce the perimeter in order to form compact and contiguous areas. These constraints evade fragmentation. Since some cells are not eligible (no agricultural land, so no EP-data available) as part of the target site, a mask indicating the availability of the cells is expressed with the set of variables *m*; therefore if the *i,j* cell is available, the variable $m_{i,j}$ will uphold a value 1, and 0 otherwise. The set of constraints in equation 9 avoid selecting unavailable cells as part of the solution, and equation 10 restricts the number of selected cells to be equal to the predefined number *M*.

3.3 Heuristic for Multiple-Criteria Site Location (HMSL)

An alternative solution method is developed to locate feasible (near to optimal) sites that fulfill multiple criteria requirements. This solution method is based on the heuristic approach developed by Church et al (2003), introducing three main differences with respect to the original process: (*1)* In order to remove the influence of value ranges, all criteria involved in the current problem, including the suitability rewarding compactness, are normalized. After this process, all raster maps representing criteria use the same measuring scale, each cell upholding values between 0 and 1. Normalization allows all criteria compete among them under the same conditions; (*2)* The proposed heuristic for site location (HMSL) makes use of objective functions considering weighted multiple criteria (MADM). In equations 16 and 17, the denominator act as a weight normalization factor. They control that the increase in importance (weight) of one criterion implies also the decrease of importance of the others; (*3)* A number of seed patches are automatically generated, and the region growing process is repeated to produce several candidate solutions.

The method developed here is a Multiple-Criteria Heuristic solution method for Site Location (HMSL), which is divided in 3 stages: *1)* seeds generation, *2)* region growing, and *3)* region ranking. The final goal of the entire algorithm (figure 1) is to construct a compact site maximizing its intrinsic multiple criteria suitability as derived from the suitability of the member cells. Environmental Performance (EP) criteria are applied to test this method as well as the mathematical formulation.

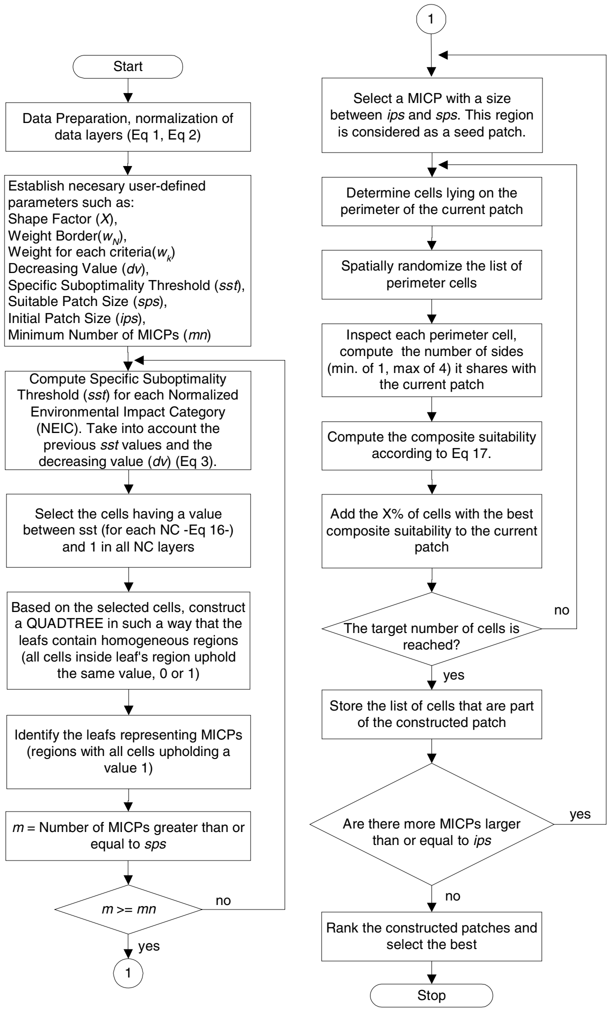

Fig. 1. General Flow Diagram of the Heuristic for Multi-Criteria Site Location (HMSL)

3.3.1 Generation of Seed Patches

The foundation of seed patch generation is with interval goal programming and on quadtree-based search. While the first deals with multiple criteria analysis, the second technique is applied for searching Maximum Initial Compact Patches (MICPs). An individual MICP is defined as a square area composed by a set of *n x n* cells.

Multiple Criteria Analysis

The first step to generate seed patches (MICPs) is selecting a set of individual cells with maximal intrinsic multiple criteria suitability. To achieve this objective, goal programming with intervals (Charnes and Collomb, 1972; Ignizio, 1974) is applied. A pixel is considered optimal when it upholds the highest levels for each Normalized Criteria (NC) considered as part of a specific decision problem. But when the maximum values do not coincide in the same pixel, suboptimal pixels need to be selected through an iterative process.

Those suboptimals are identified by a gradual increase of a suboptimality tolerance with respect to 1 (1 = maximum NC value). Therefore, a *specific suboptimality threshold (sst)* for each NC is computed, starting at an initial value 1, and iteratively decreasing in a delta value *dv*. The *sst* is computed according to equation 16, where $psst_{nc}$ refers to the previous *sst* value (started at 1), w_{nc} is the weight of a specific NC, *dv* the decreasing value, and $\sum_k w_K$ is a normalization factor summing all weights. Equation 16 shows that the weight of the NC determines how fast sst_{nc} decreases. This process allows selecting all the cells having each NC level between the corresponding *sst* value and 1 (suboptimal cells). Consequently, the increase in tolerance implies the decrease of the *specific suboptimality threshold* (*sst*). In every iteration, the multi-criteria analysis produces a map with selected cells upholding a value 1, and with unselected cells upholding a value 0. This procedure is iterated until a set of suboptimal cells (*S*) is selected, in such a way that those cells form a *minimum number* of MICPs, each one having a size greater than or equal to a predefined *suitable patch size (sps)*. Thus, if *s* is a suboptimal cell from *S*, a set $O(s) \subset S$ is a set of cells that form a MICP sizing at least *sps*. The *minimum number* of MICPs is referred as *mn* in the algorithm (figure 1), and the number of cells pertaining to each MICP measures its size. A quadtree-based search is applied in order to find the MICPs within the current selected cells.

$$sst_{nc} = psst_{nc} - \left(\frac{dv * w_{nc}}{\sum_k w_k} \right) \tag{16}$$

Quadtree Based Search of Maximum Initial Compact Patches (MICP)

In a quadtree, the nodes branch off to four children. Trees have been widely applied in search algorithms, and particularly in the present approach the quadtree allows finding homogeneous MICPs composed by cells upholding a value 1 in a binary map. While the root of the quadtree corresponds to the entire binary map, each one of the four children corresponds to one of the four quadrants within this map. Each quadrant is recursively divided in four new quadrants until all cells within a quadrant are homogeneous (all cells upholding a value either 1 or 0). Figure 2 shows an example of the process applied to build the quadtree in figure 3. The process starts using the lines marked with 1 to divide the entire region in *four* quadrants (north west -a-, north east -b-, south west -c-, south east -d-). These lines correspond to the root node in figure 3. Since 3 (out of 4) quadrants are not homogeneous, they are divided again with lines marked as 2. The homogeneous region that is not divided corresponds to the white leaf node located left most in the second level of the quadtree in figure 3. This process is iterated until quadrants are homogeneous, and do not require to be divided again. Once this recursive process ends, homogeneous quadrants must correspond to the quadtree's leaves. Each leaf linked with regions upholding a value 1 (nodes in gray in figure 3) is considered as a potential MICP. Since the entire map can be a rectangular area instead of a squared area, each recursive child can be rectangular as well. If it is the case, a MICP corresponds to the biggest square inside the leaf's rectangular area; otherwise the complete leaf's region is a MICP. Thus, the multi-criteria analysis, together with the quadtree-based search are iterated until at least *mn* MICPs are generated, with a size not smaller than the *suitable patch size (sps)* parameter.

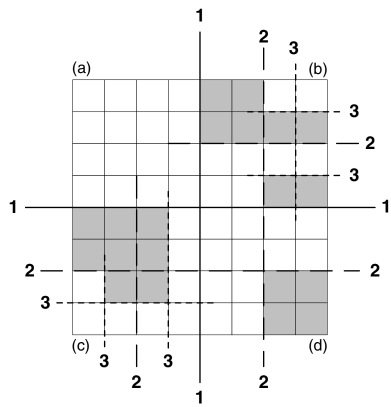

Fig. 2. Binary Image. Lines 1,2 and 3 show how the image is iteratively divided in quadrants

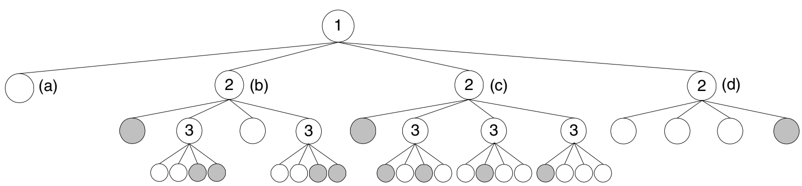

Fig. 3. Quadtree structure result of the iterative process illustrated in figure 2

3.3.2 Region Growing

Although the size of every MICP must be larger than or equal to a predefined suitable patch size (*sps*), other homogeneous regions smaller than *sps* could also be present in the quadtree's leaves. To increase the size of the search space, the region growing process considers as seeds all regions (MICPs) greater than or equal to an *initial patch size (ips)*. The parameters *ips* and *sps* control the number of candidate solutions to be created.

The algorithm proposed by Church et al (2003) is applied to each generated seed patch. As in the mathematical formulation, the Von Neumann neighborhood allows constructing a list with the cells neighboring the seed patch, and the order within this list is randomized so that the order does not play a role in the final selection of the cells to be added to the patch. Next, a composite suitability is computed for each neighboring cell, and the list is ordered according to this suitability. Although the cells with the same composite value appear as a group, they are in random order inside the group. Finally, the best X % of neighboring cells are added to the seed patch to form a new patch which is used in a next iteration to add more cells until a predefined number (M) is achieved as part of the final site. The composite suitability equation proposed by Church et al (2003) is substituted by equation 17; where: i refers to each i_{th} neighboring cell; P is the number of Normalized Criteria (NCs) considered as part of the problem; $f(k_i)$ is the k_{th} NC value for the i_{th} neighboring cell; w_k the weight associated with this NC; and e_i is the normalized value characterizing the number of edges that the i_{th} neighbouring cell shares with the current growing patch. Since the maximum number of shared edges (se) is 4, the value e_i is equal to se multiplied by a constant value of 0.25 ($e_i = se * 0.25$); the value e_i is weighted according to w_N. As in Church et al (2003), the parameters w_N and X control the growing process; X is the percentage of cells to be added to the current patch in each iteration, and w_N weights the number of edges shared with the current patch. For convenience, a slightly different notation is used here: (1) w_N stands for weight border, and is equivalent to the parameter N in Church et al (2003) and (2) X is referred as a shape factor and has the same meaning as in Church et al (2003).

$$suit_i = \frac{\sum_{k=1}^{P} w_k * f(k_i) + w_n * e_i}{\sum_{k=1}^{P} w_k + w_N} \tag{17}$$

3.3.3 Region Ranking

Once several patches (candidate solutions) have been generated, the best solution is selected through a multi-criteria ranking procedure. Each candidate solution is represented as a binary map, where cells upholding a value 1 are part of the final site, and cells with a value 0 are not. The accumulated composite suitability for every site R is computed with equation 18; M is the total number of cells that are part of the final site, P is the total number of layers (NCs) taken into account, g_i the number of edges that the i_{th} cell shares with adjacent cells upholding a value 1, and $f(k_i)$ is the normalized value for the impact category k in the cell i. As before, a constant value ($c=0.25$) is applied to normalize g_i. The accumulated composite suitability is calculated for each patch, and the one corresponding with the highest value is chosen as the best.

$$suit_R = w_N * \sum_{i=1}^{M} g_i * c + \sum_{i=1}^{M} \sum_{k=1}^{P} w_k * f(k_i) \tag{18}$$

4 Results and Discussion

In a first stage the Heuristic for Multiple-Criteria Site Location (HMSL) and the Integer Programming (IP) formulation are compared using reduced areas (*70 x 70 cells*) within the Netherlands, Denmark and Flanders. Afterwards, every entire territory is processed with the HMSL, and a sensitivity analysis is carried out to determine its behavior for different parameter values. The compactness criterion and two normalized intrinsic criteria are considered in the tests: carbon sequestration (*CS*) and nitrate leaching (*NL*) after 10 years of afforestation with oak under intensive stand preparation and intensive stand tending levels. The impact categories are normalized according to equation 1 in the case of *CS*, and according to equation 2 in the case of *NL.* The tests are carried out in a computer with a Pentium-3GHz processor and 1Gb in RAM.

4.1 Comparison of the Mathematical and Heuristic Approaches

During the comparison, the tests are configured in order to select patches of 30 cells. In both approaches, the weight border (w_N), and the weights for CS and NL are constant, the value 0.33 is assigned to each criterion. Whereas the mathematical formulation is solved in LPSolve-IDE (Gourvest et al, 2006), the heuristic solution method is implemented in Java.

The Shannon entropy (equation 19), based on the number of pixel classes (table 2) is applied to measure the amount of information in each reduced region (raster map). In equation 19, *n* is the number of pixel classes, and $p(x_l)$ is the probability of occurrence of pixel class x_l. The Shannon entropy is used here as a measure for homogeneity, or level of uniformity in the composition of the maps. The lowest amount of information corresponds to the highest homogeneity, and the highest amount of information with the opposite. Consequently, Flanders is the least homogeneous areas and Denmark the most homogeneous.

$$H(X) = -\sum_{l=1}^{n} p(x_l) * Ln(p(x_l)) \tag{19}$$

To evaluate the HMSL, 144 tests are carried out in each reduced area using different values (table 1) for shape factor (*X*), suitable patch size (*sps*), initial patch size (*ips*), and decreasing value (*dv*) (section 3.3. explains the meaning of these parameters). The weights for CS and NL, and the weight border (w_N) stay constant in 0.33, and *mn* is kept in 1. This configuration gives *6 x 3 x 8 = 144* tests for each reduced region, the results of which are summarized in table 3.

In tables 2 and 3 i the IP results are compared with the summary of the 144 tests performed with the HMSL. Regarding computation time required for reaching the solutions, the heuristic (table 3) is considerably faster than the IP model (table 2). While the mathematical approach requires time in the order of minutes and even hours as in the cases of the Netherlands and Denmark, time average of the 144 HMSL tests is around one second.

In Flanders, 29.17% of HMSL tests give solutions deviating less than 0.5% from the objective value obtained with the mathematical approach, and 99.3% deviate less than 2% (table 3). Concerning spatial location, 55% of the solutions have a centroid less than 4 cell apart from the centroid of the site generated by the mathematical approach; this percentage represents the number of patches that are very near to the mathematical objective value, but that at the same time are located in the same region where the mathematically generated site is located. Since *30* cells are selected as part of the final patch, a *4* cells distance is suitable to know whether the solutions are spatially near.

Table 1. Values for *sps-ips*, *dv* and *X* applied in the reduced tests

sps-ips	*dv*	*X*
6 - 2	0.002	0.1
8 - 2	0.01	0.2
15 - 2	0.02	0.3
		0.4
6 - 3		0.5
8 - 3		0.6
15 - 3		0.7
		0.8
		0.9

In The Netherlands, 4.17% of the tests give solutions deviating less than 0.5% from the mathematical objective value, 81.94% deviate less than 1%, and all solutions deviate less than 2%. On the other hand, 63% of HMSL tests produce a site with a centroid less than 4 cells apart from the mathematical centroid.

25.69% of HMSL solutions in Denmark deviate less than 0.5% from the objective value obtained with the mathematical approach, and all solutions deviate less than 1%. Homogeneity plays an important role locating the final patch with the heuristic approach, only 2% of the tests produce solutions having a centroid less than 4 cells apart from the optimal centroid (x=64,y=31.5; table 2). This low percentage is obtained because of the high homogeneity present in Denmark, where a huge number of near to optimal solutions likely exist.

Table 3 shows that while in Denmark, the most homogeneous region, it is easier to obtain solutions near to the optimality reference given by the IP results, the distribution of the solutions in Flanders shows that it is harder to obtain spatially near to optimal solutions. These results are in the agreement with the average of cells overlapping the mathematical solution (table 3).

A more detailed analysis of the overlapping cells can be performed with the results shown in figures 4a and 4b. Although the location and shape of the final site vary in every HMSL test case because of the random ordering of equal valued candidate cells in the *region growing* step, figure 4 gives some thoughts about the influence of *sps-isp* configurations over the results. In Flanders (figure 4a), small *sps-ips* intervals can find sites more closely located to the one from the mathematical solution when the delta interval (*dv*) is large (*0.01, 0.02*). Since Flanders is the least homogeneous region, small values of *sps* and *ips* guarantee good quality seed patches, which in turn generate the best results not only from the intrinsic suitability but also from the location point of view. Taking into account that large *sps* implies the generation of larger seed patches, not high quality cells in this heterogeneous region could be included as part of these seeds. Accordingly, larger *sps-ips* ranges do not guarantee finding best seeds and nor best solutions. Regarding the delta value (*dv*) for small *sps-ips* ranges, it is feasible to conclude that the smallest *dv* is generating less seed patches although they could be also good quality ones. Therefore the final solutions are of lower quality than the ones obtained for larger *dvs*. Nevertheless, location quality for small *dv* values are increased for large *sps-ips* intervals, where more seed patches are.

Table 2. Results obtained by the mathematical approach

Region				***Mathematical Model***			
	Size (Cells)	**Pixel Classes**	**Shannon Entropy**	**Objective Value**	**Centroid X**	**Centroid Y**	**Time (sec)**
Flanders	70 x 70	43	2.51	25.02	64.5	12	548
The Netherlands	70 x 70	117	2.41	24.77	17.5	27	28450
Denmark	70 x 70	26	2.3	26.74	64	31.5	13204

Table 3. Results obtained by the heuristic approach and comparison with the mathematical solution

Summary of the 144 Tests performed with HMSL in each Reduced Region	**Deviation from the Objective Value Obtained with the Mathematical Model (Less than)**				**HMSL Solutions less than 4 cells apart from the centroid of the mathematical solution**	**Average of cells overlapping the mathematical solution**	**Time avg. (sec)**	
	0.5%	***1.0%***	***1.5%***	***2.0%***			***Min***	***Max***
Flanders	29.17%	60.42%	95.14%	99.3%	55%	12	0.02	0.6
The Netherlands	4.17%	81.94%	95.14%	100 %	63%	15	0.3	1.5
Denmark	25.69%	100 %	100 %	100 %	2%	0	0.8	0.8

In line with the Shannon Entropy (table 2) according to which the Netherlands is more homogeneous than Flanders, the results show a more uniform behavior for larger *sps-ips* intervals (figure 4b). This is explained by the fact that although in more homogeneous areas good quality seed patches can be generated with small *sps-ips* intervals, the homogeneity condition implies generating more seed patches in order to generate higher quality solutions from the location standpoint. This argumentation is also valid for explaining the behavior of *dv* and *sps-ips* ranges in Denmark, the most homogeneous region.

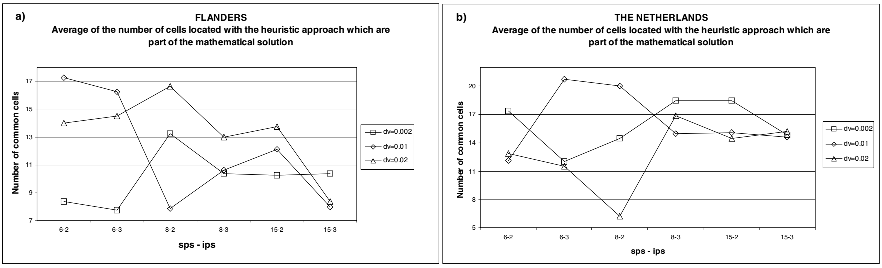

Fig. 4. Average number of common cells obtained with the heuristic and mathematical approaches

4.2 Heuristic Approach

The Heuristic method for Multiple-Criteria Site Location (HMSL) is applied to locate 3000 cells forming compact patches within Denmark, Flanders, and The Netherlands. The criteria are the same to the ones applied for the reduced areas (section 4.1): maximize Carbon Sequestration (CS) and minimize Nitrate Leaching (NL).

A sensitivity analysis is carried out to determine how the *weight border* (w_N) and the *shape factor* (X) control the heuristic site search process. Twenty-four tests are performed varying w_N with values in the set {0.2, 0.5, 0.9}, and X with values in the set {0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9}. The difference between 1 and w_N is equitably distributed for weighting CS and NL. The *Shape Index* (*SI*) in equation 20 (Maceachren, 1985) measures the compactness of each constructed site, taking values between 0 and 1, with the highest values corresponding to the highest compactness. Perimeter and area are computed taking into account the cells part of the final patch. On the other hand, the *Intrinsic* or *Core Suitability* (*CoS*) of the patches is measured according to equation 21, where $f(k_i)$ is the k_{th} NC value in the i_{th} cell part of the final site. The heuristic sensitivity analysis is shown in figures 4, 5, and 6. The sub-figures (a) show the *Core Suitability* (*CoS*) and the sub-figures (b) show the *Shape Index* (*SI*) of the sites obtained in the Netherlands, Flanders, and Denmark.

$$SI = \frac{\sqrt{area}}{0.282 * perimeter} \tag{20}$$

$$CoS = \sum_{i=1}^{M} \sum_{k=1}^{P} f(k_i) \tag{21}$$

As in the reduced tests, the Shannon Entropy measures the amount of information in each entire region. According to the measures summarized in table 4, Flanders has the highest amount of information (highest heterogeneity), and Denmark the lowest (highest homogeneity).

Table 4. Shannon Entroy in The Netherlands, Flanders and Denmark

Region	Original Resolution	Size (Cells)	Pixel Classes	Shannon Entropy
Flanders	100m x 100m	1000 x 2500	1072	2.36
The Netherlands	250m x 250m	1268 x 1076	1742	2.23
Denmark	100m x 100m	3590 x 4740	1413	1.30

Regarding the influence of the Shape Factor parameter (X), figures 5 to 7 show a common behavior, sites with the highest Shape Index (SI) and the highest accumulated Core Suitability (CoS) are obtained with the smallest X values. Since a low X guarantees that the best pixels are added to the current patch, large X values will result in the addition of sub-optimal pixels. This characteristic of the HMSL explains the lack of clarity in the influence of w_N for X values larger than 0.6.

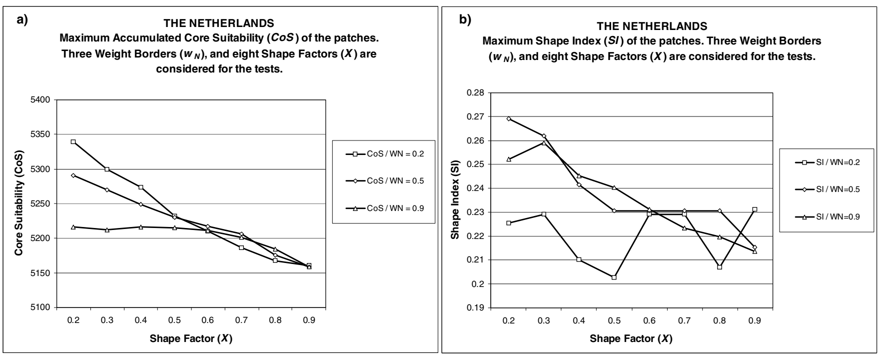

Fig. 5. The Netherlands: Test results for three Weights for Border (W_N) and eight Shape Factors (X). a) Core Suitability; b) Shape Index.

Figures 5 to 7 also show that while SI is favored by higher values of w_N, CoS is favored by lower values of w_N. This behavior is in agreement with the logic of the region-growing algorithm, where w_N is in competition with the weights applied to the criteria involved in the CoS computation (Carbon Sequestration and Nitrate Leaching in the tests). There is also evidence that w_N has a substantial impact for the CoS in Denmark and in The Netherlands. In the tests for these regions (figures 6a and 6b), the differences in CoS introduced by w_N are clearly identified. Indeed the differences are most consistent in Denmark, the most homogeneous area (lowest Shannon Entropy -table 4-).

On the other hand, the influence of w_N is less apparent in Flanders (figures 6a, 6b) which is the region with the lowest homogeneity. Therefore, it is feasible to assert that w_N has a higher significance for CoS in more homogeneous regions. This can be explained by the fact that in homogeneous regions the candidate pixels to be included as part of the patch are more or less similar from the intrinsic suitability point of view (*CoS*), which implies that, the best pixels are strongly supported by w_N. The opposite behavior is found in less homogeneous areas (e.g. Flanders).

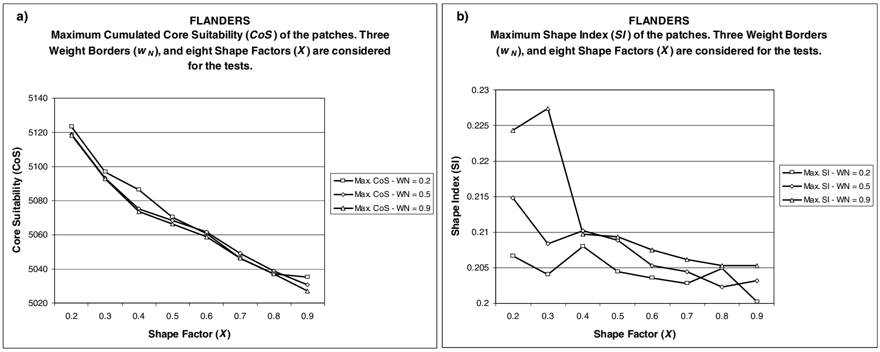

Fig. 6. Flanders: Test results for three Weights for Border (W_N) and eight Shape Factors (X). a) Core Suitability; b) Shape Index.

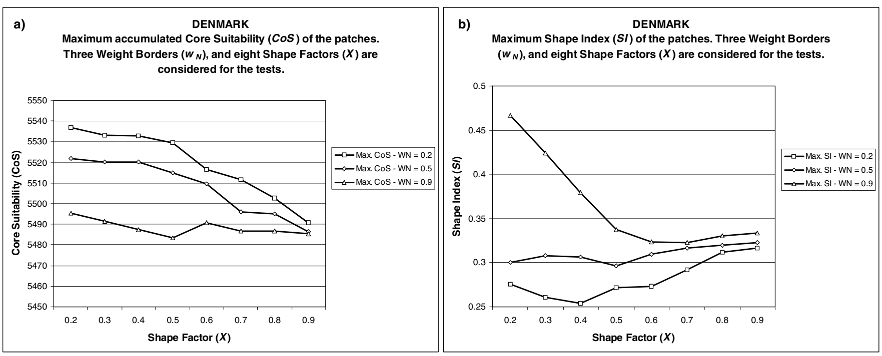

Fig. 7. Denmark: tests results for three Weights for Border (W_N) and eight Shape Factors (X). a) Core Suitability; b) Shape Index.

Figure 8 shows the average of computation time required for the proposed heuristic in order to complete the region-growing process for an individual seed patch. Time required for building a complete site depends on the number of cells required to form it, as well as on the X parameter. Figure 8a shows that the data follow a power model, in which computing time decreases exponentially as X increases.

Data in figure 8b correspond to the computation time for growing different sized sites (from 200 to 3000 cells) in the three testing regions, but keeping $X=0.2$. These results also follow a power model, where computation time increases with the number of cells. Figure 8b shows that computation time for growing patches in Denmark is slightly higher than times recorded for the Netherlands and Flanders.

Since these small differences are produced when the final site is composed by a high number of cells, this behavior can have an explanation in the performance of the sorting algorithm included as part of the region-growing process. The results show that the sorting algorithm likely has inferior performance for the regions with the lowest Shannon entropy. The differences are not substantial though.

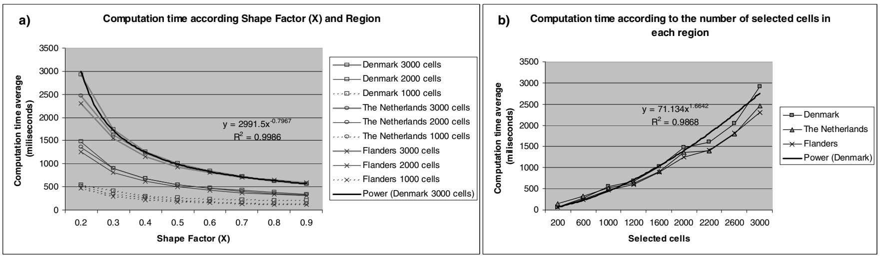

Fig. 8. Average of computing time for the region-growing process included as part of the proposed heuristic. a) Time according to Shape Index (X), number of cells, and region. b) Time according to region, and number of selected cells, keeping $X=0.2$.

Figure 9 shows the most suitable site of 3000 cells generated with HMSL assigning the same weight to compactness as well as to two intrinsic criteria: Carbon Sequestration (*CS*) and Nitrate Leaching (*NL*). As shown in figure 9, the site is composed by pixels maximizing *CS* (pixels in dark gray) and minimizing *NL* (pixels in bright gray).

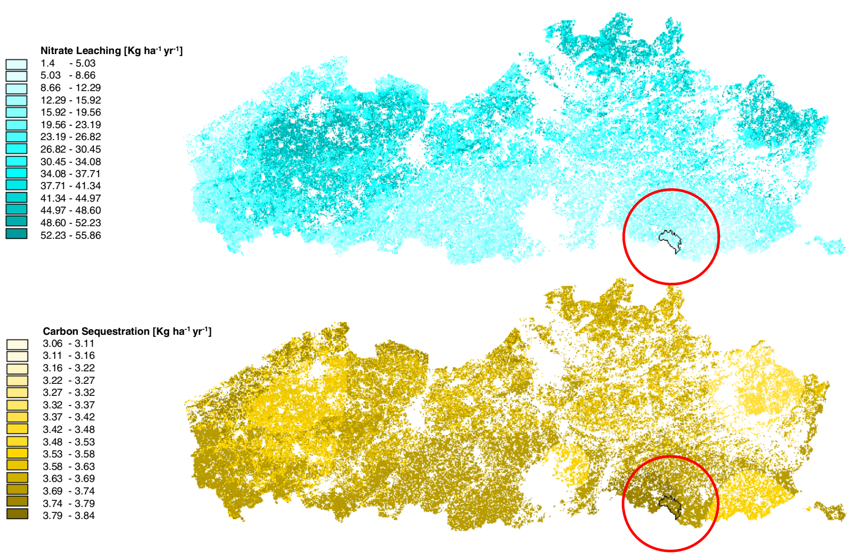

Fig. 9. Most suitable site of 3000 cells for afforestation with oak in Flanders

5 Conclusions and Further Work

Two methods were described to generate compact sites maximizing the intrinsic or core multiple criteria suitability of the set of cells forming these sites: a reference mathematical method and a heuristic approach (HMSL). Both approaches were compared using datasets of limited size. It was found that the HMSL heuristic finds solutions which are very much in line with the mathematical solutions. According to the tests, many solutions near to the optimal objective value can exist, and these solutions can even be spatially distributed. Nevertheless, most of the heuristic tests generate solutions that are also spatially near to the optimal patch location. The comparison also revealed that although the suitable patch size (*sps*), and the initial patch size (*ips*) parameters control the size of the search space, large search spaces (large *sps*, small *ips*) do not necessarily improve the quality of the solutions. Selection of *sps-ips* ranges should consider the level of homogeneity of the information at hand knowing that search space size has a direct impact on computation time.

HMSL tests in real sized data for growing one site of 3000 cells and keeping $X=0.2$, required a computation time around 3 seconds. These results show that HMSL is entirely applicable for real sized problems, even considering a high number of candidate solutions. The similarity of the results with respect to the mathematical approach, and the low computation time allow concluding that HMSL has a high performance.

The sensitivity analysis shows that the parameter X (percentage of cells added to the growing patch in each iteration) has a predominant influence on the computation time and on the quality of the results. Low values for X guarantee high quality solutions. Values lower than 0.6 are seem to be adequate for this parameter since this value range does not only guarantee near to optimal solutions, but also allows keeping a clear behavior of the heuristic when w_N (weight given to the compactness criterion) is changed. On the other hand, high values in w_N generate more compact areas, while low values increase core suitability.

Taking the Shannon entropy as a homogeneity approximation, it is feasible to assert that the heuristic is more sensitive to changes of criteria weights in areas with less information (more homogeneous), and for X values lower than 0.6.

HMSL can be improved by including goal programming with intervals, as part of the region growing stage. This improvement will guarantee that each pixel added to the current patch is effectively the best for one or more criteria. The computation time will rise though.

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